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# THE MONIST

## THE PHILOSOPHY OF BERGSON.[1]

### I.

THE classification of philosophies is effected, as a rule, either by their methods or by their results: "empirical" and "*a priori*" is a classification by methods, "realist" and "idealist" is a classification by results. An attempt to classify Bergson's philosophy in either of these ways is hardly likely to be successful, since it cuts across all the recognized divisions.

But there is another way of classifying philosophies, less precise, but perhaps more helpful to the non-philosophical; in this way, the principle of division is according to the predominant desire which has led the philosopher to philosophize. Thus we shall have philosophies of feeling, inspired by the love of happiness; theoretical philosophies, inspired by the love of knowledge; and practical philosophies, inspired by the love of action.

Among philosophies of feeling we shall place all those which are primarily optimistic or pessimistic, all those that offer schemes of salvation or try to prove that salvation is impossible; to this class belong most religious philosophies. Among theoretical philosophies we shall place most of the great systems; for though the desire for knowledge is rare, it has been the source of most of what is best in philosophy. Practical philosophies, on the other hand, will be those

[1] The abbreviations of the titles of the works of M. Bergson referred to are: *C. E., Creative Evolution; M. and M., Matter and Memory; T and F. W., Time and Free Will.* The references are to the English translations of M. Bergson's books.

which regard action as the supreme good, considering happiness an effect and knowledge a mere instrument of successful activity. Philosophies of this type would have been common among Western Europeans if philosophers had been average men; as it is, they have been rare until recent times, in fact their chief representatives are the pragmatists and M. Bergson. In the rise of this type of philosophy we may see, as M. Bergson himself does, the revolt of the modern man of action against the authority of Greece, and more particularly of Plato; or we may connect it, as Dr. Schiller apparently would, with imperialism and the motor-car. The modern world calls for such a philosophy, and the success which it has achieved is therefore not surprising.

M. Bergson's philosophy, unlike most of the systems of the past, is dualistic: the world, for him, is divided into two disparate portions, on the one hand life, on the other matter, or rather that inert something which the intellect views as matter. The whole universe is the clash and conflict of two opposite motions: life, which climbs upward, and matter, which falls downward. Life is one great force, one vast vital impulse, given once for all from the beginning of the world, meeting the resistance of matter, struggling to break a way through matter, learning gradually to use matter by means of organization; divided by the obstacles it encounters into diverging currents, like the wind at the street-corner; partly subdued by matter through the very adaptations which matter forces upon it; yet retaining always its capacity for free activity, struggling always to find new outlets, seeking always for greater liberty of movement amid the opposing walls of matter.

Evolution is not primarily explicable by adaptation to environment; adaptation explains only the turns and twists of evolution, like the windings of a road approaching a town through hilly country. But this simile is not quite

adequate; there is no town, no definite goal, at the end of the road along which evolution travels. Mechanism and teleology suffer from the same defect: both suppose that there is no essential novelty in the world. Mechanism regards the future as implicit in the past, since it believes the future to be calculable; teleology also, since it believes that the end to be achieved can be known in advance, denies that any essential novelty is contained in the result.

As against both these views, though with more sympathy for teleology than for mechanism, M. Bergson maintains that evolution is truly *creative*, like the work of an artist. An impulse to action, an undefined want, exists beforehand, but until the want is satisfied it is impossible to know the nature of what will satisfy it. For example, we may suppose some vague desire in sightless animals to be able to be aware of objects before they were in contact with them. This led to efforts which finally resulted in the creation of eyes. Sight satisfied the desire, but could not have been imagined beforehand. For this reason, evolution is unpredictable, and determinism cannot refute the advocates of free will.

This broad outline is filled in by an account of the actual development of life on the earth. The first division of the current was into plants and animals: plants aimed at storing up energy in a reservoir, animals aimed at using energy for sudden and rapid movements. "The same impetus," he says, "that has led the animal to give itself nerves and nerve centers must have ended, in the plant, in the chlorophyllian function" (*C. E.*, p. 120). But among animals, at a later stage, a new bifurcation appeared: *instinct* and *intellect* became more or less separated. They are never wholly without each other, but in the main intellect is the misfortune of man, while instinct is seen at its best in ants, bees, and Bergson. The division between intellect and instinct is fundamental in his philosophy, much

of which is a kind of Sandford and Merton, with instinct as the good boy and intellect as the bad boy.

Instinct at its best is called *intuition*. "By *intuition*," he says, "I mean instinct that has become disinterested, selfconscious, capable of reflecting upon its object and of enlarging it indefinitely" (*C. E.*, p. 186). The account of the doings of intellect is not always easy to follow, but if we are to understand Bergson we must do our best.

Intelligence or intellect, "as it leaves the hands of nature, has for its chief object the inorganic solid" (*C. E.*, p. 162); it can only form a clear idea of the discontinuous and the immobile (pp. 163-4); its concepts are outside each other like objects in space, and have the same stability (p. 169). The intellect separates in space and fixes in time; it is not made to think evolution, but represent *becoming* as a series of states (p. 171). "The intellect is characterized by a natural inability to understand life" (p. 174); geometry and logic, which are its typical products, are strictly applicable to solid bodies, but elsewhere reasoning must be checked by common sense, which, as Bergson truly says, is a very different thing (p. 170). Solid bodies, it would seem, are something which mind has created on purpose to apply intellect to them, much as it has created chess-boards in order to play chess on them. The genesis of intellect and the genesis of material bodies, we are told, are correlative: both have been developed by reciprocal adaptation (p. 196). "An identical process must have cut out matter and the intellect, at the same time, from a stuff that contained both" (p. 210).

This conception of the simultaneous growth of matter and intellect is ingenious, and deserves to be understood. Broadly, I think, what is meant is this: Intellect is the power of seeing things as separate one from another, and matter is that which is separated into distinct things. In reality there are no separate solid things, only an endless

stream of becoming, in which nothing becomes and there is nothing that this nothing becomes. But becoming may be a movement up or a movement down: when it is a movement up it is called life, when it is a movement down it is what, as misapprehended by the intellect, is called matter. I suppose the universe is shaped like a cone, with the Absolute at the vertex, for the movement up brings things together, while the movement down separates them, or at least seems to do so. In order that the upward motion of mind may be able to thread its way through the downward motion of the falling bodies which hail upon it, it must be able to cut out paths between them; thus as intelligence was formed, outlines and paths appeared (p. 199), and the primitive flux was cut up into separate bodies. The intellect may be compared to a carver, but it has the peculiarity of imagining that the chicken always was the separate pieces into which the carving-knife divides it.

"The intellect," Bergson says, "always behaves as if it were fascinated by the contemplation of inert matter. It is life looking outward, putting itself outside itself, adopting the ways of inorganized nature in principle, in order to direct them in fact" (p. 170). If we may be allowed to add another image to the many by which Bergson's philosophy is illustrated, we may say that the universe is a vast funicular railway, in which life is the train that goes up, and matter is the train that goes down. The intellect consists in watching the descending train as it passes the ascending train in which we are. The obviously nobler faculty which concentrates its attention on our own train, is instinct or intuition. It is possible to leap from one train to the other; this happens when we become the victims of automatic habit, and is the essence of the comic. Or we can divide ourselves into parts, one part going up and one down; then only the part going down is comic. But intellect is not itself a descending motion, it is merely an

observation of the descending motion by the ascending motion.

Intellect, which separates things, is, according to Bergson, a kind of dream; it is not *active*, as all our life ought to be, but purely contemplative. When we dream, he says, our self is scattered, our past is broken into fragments (p. 212),[2] things which really interpenetrate each other are seen as separate solid units: the extra-spatial degrades itself into spatiality (p. 218), which is nothing but separateness. Thus all intellect, since it separates, tends to geometry, and logic, which deals with concepts that lie wholly outside each other, is really an outcome of geometry, following the direction of materiality (pages 222-4). Both deduction and induction require spatial intuition behind them (p. 225); "the movement at the end of which is spatiality lays down along its course the faculty of induction, as well as that of deduction, in fact, intellectuality entire." It creates them in mind, and also the order in things which the intellect finds there (p. 228). Thus logic and mathematics do not represent a positive spiritual effort (p. 224), but a mere somnambulism, in which the will is suspended, and the mind is no longer active. Incapacity for mathematics is therefore a sign of grace—fortunately a very common one.

As intellect is connected with space, so instinct or intuition is connected with time. It is one of the noteworthy features of Bergson's philosophy that, unlike most writers, he regards time and space as profoundly dissimilar. Space, the characteristic of matter, arises from a dissection of the flux which is really illusory, useful, up to a certain point, in practice, but utterly misleading in theory. Time, on the contrary, is the essential characteristic of life or mind. "Wherever anything lives," he says, "there is, open some-

[2] It is noteworthy that elsewhere Bergson speaks of dreams as giving us duration more pure than in waking life (*T. and F. W.*, p. 126).

where, a register in which time is being inscribed" (*C. E.*, p. 17). But the time here spoken of is not mathematical time, the homogeneous assemblage of mutually external instants. Mathematical time, according to Bergson, is really a form of space; the time which is of the essence of life is what he calls *duration.* This conception of duration is fundamental in his philosophy; it appears already in his earliest book *Time and Free Will,* and it is necessary to understand it if we are to have any comprehension of his system. It is, however, a very difficult conception. I do not fully understand it myself, and therefore I cannot hope to explain it with all the lucidity which it doubtless deserves.

"Pure duration," we are told, "is the form which our conscious states assume when our ego lets itself *live,* when it refrains from separating its present state from its former states" (*T. and F. W.,* p. 100). It forms the past and the present into one organic whole, where there is mutual penetration, succession without distinction (*ib.*). "Within our ego, there is succession without mutual externality; outside the ego, in pure space, there is mutual externality without succession" (p. 108).

"Questions relating to subject and object, to their distinction and their union, should be put in terms of time rather than of space" (*M. and M.*, p. 77). In the duration in which we *see ourselves acting,* there are dissociated elements; but in the duration in which we *act,* our states melt into each other (*M. and M.,* p. 243). Pure duration is what is most removed from externality and least penetrated with externality, a duration in which the past is big with a present absolutely new. But then our will is strained to the utmost; we have to gather up the past which is slipping away, and thrust it whole and undivided into the present. At such moments we truly possess ourselves, but such moments are rare (*C. E.,* pp. 210-211). Duration is

the very stuff of reality, which is perpetual becoming, never something made (*C. E.*, p. 287).

It is above all in *memory* that duration exhibits itself, for in memory the past survives in the present. Thus the theory of memory becomes of great importance in Bergson's philosophy. *Matter and Memory* is concerned to show the relation of mind and matter, of which both are affirmed to be real (p. vii), by an analysis of memory, which is "just the intersection of mind and matter" (p.xii).

There are, to begin with, two radically different things, both of which are commonly called *memory*; the clear distinction between these two is one of the best things in Bergson. "The past survives," he says, "under two distinct forms: first, in motor mechanisms; secondly, in independent recollections" (*M. and M.*, p. 87). For example, a man is said to remember a poem if he can repeat it by heart, that is to say, if he has acquired a certain habit or mechanism enabling him to repeat a former action. But he might, at least theoretically, be able to repeat the poem without any recollection of the previous occasions on which he has read it; thus there is no consciousness of past events involved in this sort of memory. The second sort, which alone really deserves to be called memory, is exhibited in recollections of separate occasions when he has read the poem, each unique and with a date. Here there can be no question of *habit,* since each event only occurred once, and had to make its impression immediately. It is suggested that in some way everything that has happened to us is remembered, but as a rule, only what is useful comes into consciousness. Apparent failures of memory, it is argued, are not really failures of the mental part of memory, but of the motor mechanism for bringing memory into action. This view is supported by a discussion of brain physiology and the facts of amnesia, from which it is held to result that true memory is not a function of the brain (*M. and M.*,

p. 315). The past must be *acted* by matter, *imagined* by mind (*M. and M.*, p. 298). Memory is not an emanation of matter; indeed the contrary would be nearer the truth if we mean matter as grasped in concrete perception, which always occupies a certain duration (*M. and M.*, p. 237).

"Memory must be, in principle, a power absolutely independent of matter. If, then, spirit is a reality, it is here, in the phenomena of memory, that we may come into touch with it experimentally" (*M. and M.*, p. 81).

At the opposite end from pure memory Bergson places pure perception, in regard to which he adopts an ultra-realist position. "In pure perception," he says, "we are actually placed outside ourselves, we touch the reality of the object in an immediate intuition" (p. 84). So completely does he identify perception with its object that he almost refuses to call it mental at all. "Pure perception," he says, "which is the lowest degree of mind—mind without memory—is really part of matter, as we understand matter" (*M. and M.*, p. 297). Pure perception is constituted by dawning action, its actuality lies in its activity (*M. and M.*, p. 74). It is in this way that the *brain* becomes relevant to perception, for the brain is not an instrument of representation, but an instrument of action (*M. anl M.*, p. 83). The function of the brain is to limit our mental life to what is practically useful. But for the brain, one gathers, everything would be perceived, but in fact we only perceive what interests us (cf. *M. and M.*, p. 34). "The body, always turned towards action, has for its essential function to limit, with a view to action, the life of the spirit" (*M. and M.*, p. 233). It is, in fact, an instrument of choice.

We must now return to the subject of instinct or intuition, as opposed to intellect. It was necessary first to give some account of duration and memory, since Bergson's theories of duration and memory are presupposed in his

account of intuition. In man, as he now exists, intuition is the fringe or penumbra of intellect: it has been thrust out of the center by being less useful in action than intellect, but it has deeper uses which make it desirable to bring it back into greater prominence. Bergson wishes to make intellect "turn inwards on itself, and awaken the potentialities of intuition which still slumber within it" (*C. E.*, p. 192). The relation between instinct and intellect is compared to that between sight and touch. Intellect, we are told, will not give knowledge of things at a distance; indeed the function of science is said to be to explain all perceptions in terms of touch.

"Instinct alone, he says, "is knowledge at a distance. It has the same relation to intelligence that vision has to touch" (*C. E.*, p. 177). We may observe in passing that, as appears in many passages, Bergson is a strong visualizer, whose thought is always conducted by means of visual images. Many things which he declares to be necessities of all thought are, I believe, characteristic of visualizers, and would not be true of those who think by means of auditory images. He always exalts the sense of sight at the expense of the other senses, and his views on space would seem to be largely determined by this fact. I shall return to this question at a later stage.

The essential characteristic of intuition is that it does not divide the world into separate things, as the intellect does; although Bergson does not use these words, we might describe it as synthetic rather than analytic. It apprehends a multiplicity, but a multiplicity of interpenetrating processes, not of spatially external bodies. There are in truth no *things*: "things and states are only views, taken by our mind, of becoming. There are no things, there are only actions" (*C. E.*, p. 261). This view of the world, which appears difficult and unnatural to intellect, is easy and natural to intuition. Memory affords an instance of what

is meant, for in memory the past lives on into the present and interpenetrates it. Apart from mind, the world would be perpetually dying and being born again; the past would have no reality, and therefore there would be no past. It is memory, with its correlative desire, that makes the past and the future real and therefore creates true duration and true time. Intuition alone can understand this mingling of past and future: to the intellect they remain external, spatially external as it were, to one another. Under the guidance of intuition, we perceive that "form is only a snapshot view of a transition" (*C. E.*, p. 319), and the philosopher "will see the material world melt back into a single flux" (*C. E.*, p. 390).

Closely connected with the merits of intuition is Bergson's doctrine of freedom and his praise of action. "In reality," he says, "a living being is a center of action. It represents a certain sum of contingency entering into the world, that is to say, a certain quantity of possible action" (*C. E.*, p. 276). The arguments against free will depend partly upon assuming that the intensity of psychical states is a *quantity*, capable, at least in theory, of numerical measurement; this view Bergson undertakes to refute in the first chapter of *Time and Free Will*. Partly the determinist depends, we are told, upon a confusion between true duration and mathematical time, which Bergson regards as really a form of space. Partly, again, the determinist rests his case upon the unwarranted assumption that, when the state of the brain is given, the state of the mind is theoretically determinate. Bergson is willing to admit that the converse is true, that is to say, that the state of brain is determinate when the state of mind is given, but he regards the mind as more differentiated than the brain, and therefore holds that many different states of mind may correspond to one state of brain. He concludes that real freedom is possible: "We are free when our acts spring from

our whole personality, when they express it, when they have that indefinable resemblance to it which one sometimes finds between the artist and his work" (*T. and F. W.*, p. 172).

In the above outline, I have in the main endeavored merely to state Bergson's views, without giving the reasons adduced by him in favor of their truth. This is easier than it would be with most philosophers, since as a rule he does not give reasons for his opinions, but relies on their inherent attractiveness, and on the charm of an excellent style. Like the advertisers of Oxo, he relies upon picturesque and varied statement, and an apparent explanation of many obscure facts. Analogies and similes, especially form a very large part of the whole process by which he recommends his views to the reader. The number of similes for life to be found in his works exceeds the number in any poet known to me. Life, he says, is like a shell bursting into fragments which are again shells (*C. E.*, p. 103). It is like a sheaf (*ib.*, p. 104). Initially, it was "a tendency to accumulate in a reservoir, as do especially the green parts of vegetables" (*ib.*, p. 260). But the reservoir is to be filled with boiling water from which steam is issuing; "jets must be gushing out unceasingly, of which each, falling back, is a world" (*ib.*, p. 261). Again "life appears in its entirety as an immense wave which, starting from a center, spreads outwards, and which on almost the whole of its circumference is stopped and converted into oscillation: at one single point the obstacle has been forced, the impulsion has passed freely" (*ib.*, p. 280). Then there is the great climax in which life is compared to a cavalry charge. "All organized beings, from the humblest to the highest, from the first origins of life to the time in which we are, and in all places as in all times, do but evidence a single impulsion, the inverse of the movement of matter, and in itself indivisible. All the living hold together, and

all yield to the same tremendous push. The animal takes its stand on the plant, man bestrides animality, and the whole of humanity, in space and in time, is one immense army galloping beside and before and behind each of us in an overwhelming charge able to beat down every resistance and to clear many obstacles, perhaps even death" (*C. E.*, pp. 285-6).

But a cool critic, who feels himself a mere spectator, perhaps an unsympathetic spectator, of the charge in which man is mounted upon animality, may be inclined to think that calm and careful thought is hardly compatible with this form of exercise. When he is told that thought is a mere means of action, the mere impulse to avoid obstacles in the field, he may feel that such a view is becoming in a cavalry officer, but not in a philosopher, whose business, after all, is with thought: he may feel that in the passion and noise of violent motion there is no room for the fainter music of reason, no leisure for the disinterested contemplation in which greatness is sought, not by turbulence, but by the greatness of the universe which is mirrored. In that case, he may be tempted to ask whether there are any reasons for accepting such a restless view of the world. And if he asks this question, he will find, if I am not mistaken, that there is no reason whatever for accepting this view, either in the universe or in the writings of M. Bergson.

II.

The two foundations of Bergson's philosophy, in so far as it is more than an imaginative and poetic view of the world, are his doctrines of space and time. His doctrine of space is required for his condemnation of the intellect, and if he fails in his condemnation of the intellect, the intellect will succeed in its condemnation of him, for between the two it is war to the knife. His doctrine of time is

necessary for his vindication of freedom, for his escape from what William James called a "block universe," for his doctrine of a perpetual flux in which there is nothing that flows, and for his whole account of the relations between mind and matter. It will be well, therefore, in criticism, to concentrate on these two doctrines. If they are true, such minor errors and inconsistencies as no philosopher escapes would not greatly matter, while if they are false, nothing remains except an imaginative epic, to be judged on esthetic rather than on intellectual grounds. I shall begin with the theory of space, as being the simpler of the two.

Bergson's theory of space occurs fully and explicitly in his *Time and Free Will,* and therefore belongs to the oldest parts of his philosophy. In his first chapter, he contends that *greater* and *less* imply space, since he regards the greater as essentially that which *contains* the less. He offers no arguments whatever, either good or bad, in favor of this view; he merely exclaims, as though he were giving an obvious *reductio ad absurdum*: "As if one could still speak of magnitude where there is neither multiplicity nor space!" (p. 9). The obvious cases to the contrary, such as pleasure and pain, afford him much difficulty, yet he never doubts or re-examines the dogma with which he starts.

In his next chapter, he maintains the same thesis as regards number. "As soon as we wish to picture *number* to ourselves," he says, "and not merely figures or words, we are compelled to have recourse to an extended image" (p. 78), and "every clear idea of number implies a visual image in space" (p. 79). These two sentences suffice to show, as I shall try to prove, that Bergson does not know what number is, and has himself no clear idea of it. This is shown also by his definition: "Number may be defined in general as a collection of units, or, speaking more exactly, as the synthesis of the one and the many" (p. 75).

In discussing these statements, I must ask the reader's patience for a moment while I call attention to some distinctions which may at first appear pedantic, but are really vital. There are three entirely different things which are confused by Bergson in the above statements, namely: (1) number, the general concept applicable to the various particular numbers; (2) the various particular numbers; (3) the various collections to which the various particular numbers are applicable. It is this last that is defined by Bergson when he says that number is a collection of units. The twelve apostles, the twelve tribes of Israel, the twelve months, the twelve signs of the zodiac, are all collections of units, yet no one of them is the number 12, still less is it number in general, as by the above definition it ought to be. The number 12, obviously, is something which all these collections have in common, but which they do not have in common with other collections, such as cricket elevens. Hence the number 12 is neither a collection of twelve terms, nor is it something which all collections have in common; and number in general is a property of 12 or 11 or any other number, but not of the various collections that have twelve terms or eleven terms.

Hence when, following Bergson's advice, we "have recourse to an extended image" and picture, say, twelve dots such as are obtained by throwing double sixes at dice, we have still not obtained a picture of the number 12. The number 12, in fact, is something more abstract than any picture. Before we can be said to have any understanding of the number 12, we must know what different collections of twelve units have in common, and this is something which cannot be pictured because it is abstract. Bergson only succeeds in making his theory of number plausible by confusing a particular collection with the number of its terms, and this again with number in general.

The confusion is the same as if we confused a particular

young man with youth, and youth with the general concept "period of human life," and were then to argue that because a young man has two legs, youth must have two legs, and the general concept "period of human life" must have two legs. The confusion is important because, as soon as it is perceived, the theory that number or particular numbers can be pictured in space is seen to be untenable. This not only disproves Bergson's theory as to number, but also his more general theory that all abstract ideas and all logic are derived from space; for the abstract 12, the common property of all dozens as opposed to any particular dozen, though it is never present to his mind, is obviously conceivable and obviously capable of being pictured in space.

But apart from the question of numbers, shall we admit Bergson's contention that every plurality of separate units involves space? Some of the cases that appear to contradict this view are considered by him, for example successive sounds. When we hear the steps of a passer-by in the street, he says, we visualize his successive positions; when we hear the strokes of a bell, we either picture it swinging backwards and forwards, or we range the successive sounds in an ideal space (*T. and F. W.*, p. 86). But these are mere autobiographical observations of a visualizer, and illustrate the remark we made before, that Bergson's views depend upon the predominance of the sense of sight in him. There is no logical necessity to range the strokes of a clock in an imaginary space: most people, I imagine, count them without any spatial auxiliary. Yet no reason is alleged by Bergson for the view that space is necessary. He assumes this as obvious, and proceeds at once to apply it to the case of times. Where there seem to be different times outside each other, he says, the times are pictured as spread out in space; in real time, such as is given by memory, different times interpenetrate each other, and cannot be counted because they are not separate.

The view that all separateness implies space is now supposed established, and is used deductively to prove that space is involved wherever there is obviously separateness, however little other reason there may be for suspecting such a thing. Thus abstract ideas, for example, obviously exclude each other: whiteness is different from blackness, health is different from sickness, folly is different from wisdom. Hence all abstract ideas involve space; and therefore logic, which uses abstract ideas, is an offshot of geometry, and the whole of the intellect depends upon a supposed habit of picturing things side by side in space. This conclusion, upon which Bergson's whole condemnation of the intellect rests, is based, so far as can be discovered, entirely upon a personal idiosyncrasy mistaken for a necessity of thought, I mean the idiosyncrasy of visualizing successions as spread out on a line. The instance of numbers shows that, if Bergson were in the right, we could never have attained to the abstract ideas which are supposed to be thus impregnated with space; and conversely, the fact that we can understand abstract ideas (as opposed to particular things which exemplify them) seems sufficient to prove that he is wrong in regarding the intellect as impregnated with space.

One of the bad effects of an anti-intellectual philosophy, such as that of Bergson, is that it thrives upon the errors and confusions of the intellect. Hence it is led to prefer bad thinking to good, to declare every momentary difficulty insoluble, and to regard every foolish mistake as revealing the bankruptcy of intellect and the triumph of intuition. There are in Bergson's works many allusions to mathematics and science, and to a careless reader these allusions may seem to strengthen his philosophy greatly. As regards science, especially biology and physiology, I am not competent to criticize his interpretations. But as regards mathematics, he has deliberately preferred traditional er-

rors in interpretation to the more modern views which have prevailed among mathematicians for the last half century. In this matter, he has followed the example of most philosophers. In the eighteenth and early nineteenth centuries, the infinitesimal calculus, though well developed as a method, was supported, as regards its foundations, by many fallacies and much confused thinking. Hegel and his followers seized upon these fallacies and confusions, to support them in their attempt to prove all mathematics self-contradictory. Thence the Hegelian account of these matters passed into the current thought of philosophers, where it has remained long after the mathematicians have removed all the difficulties upon which the philosophers rely. And so long as the main object of philosophers is to show that nothing can be learned by patience and detailed thinking, but that we ought rather to worship the prejudices of the ignorant under the title of "reason" if we are Hegelians, or of "intuition" if we are Bergsonians, so long philosophers will take care to remain ignorant of what mathematicians have done to remove the errors by which Hegel profited.

Apart from the question of number, which we have already considered, the chief point at which Bergson touches mathematics is his rejection of what he calls the "cinematographic" representation of the world. Mathematics conceives change, even continuous change, as constituted by a series of states; Bergson, on the contrary, contends that no series of states can represent what is continuous, and that in change a thing is never in any state at all. This view that change is constituted by a series of changing states he calls cinematographic; this view, he says, is natural to the intellect, but is radically vicious. True change can only be explained by true duration; it involves an interpenetration of past and present, not a mathematical succession of static states. This is what is called a "dynamic"

instead of a "static" view of the world. The question is important, and in spite of its difficulty we cannot pass it by.

Bergson's position is illustrated—and what is to be said in criticism may also be aptly illustrated—by Zeno's argument of the arrow. Zeno argues that, since the arrow at each moment simply is where it is, therefore the arrow in its flight is always at rest. At first sight, this argument may not appear a very powerful one. Of course, it will be said, the arrow is where it is at one moment, but at another moment it is somewhere else, and this is just what constitutes motion. Certain difficulties, it is true, arise out of the continuity of motion, if we insist upon assuming that motion is also discontinuous. These difficulties, thus obtained, have long been part of the stock-in-trade of philosophers. But if, with the mathematicians, we avoid the assumption that motion is also discontinuous, we shall not fall into the philosopher's difficulties. A cinematograph in which there are an infinite number of films, and in which there is never a *next* film because an infinite number come between any two, will perfectly represent a continuous motion. Wherein, then, lies the force of Zeno's argument?

Zeno belonged to the Eleatic school, whose object was to prove that there could be no such thing as change. The natural view to take of the world is that there are *things* which *change*; for example, there is an arrow which is now here, now there. By bisection of this view, philosophers have developed two paradoxes. The Eleatics said that there were things but no changes; Heraclitus and Bergson said that there were changes but no things. The Eleatics said there was an arrow, but no flight; Heraclitus and Bergson said there was a flight but no arrow. Each party conducted its argument by refutation of the other party. How ridiculous to say there is no arrow! say the "static" party. How ridiculous to say there is no flight! say the

"dynamic" party. The unfortunate man who stands in the middle and maintains that there is both the arrow and its flight is assumed by the disputants to deny both; he is therefore pierced, like St. Sebastian, by the arrow from one side and by its flight from the other. But we have still not discovered wherein lies the force of Zeno's argument.

Zeno assumes, tacitly, the essence of the Bergsonian theory of change. That is to say, he assumes that when a thing is in a process of continuous change, even if it is only change of position, there must be in the thing some internal *state* of change. The thing must, at each instant, be intrinsically different from what it would be if it were not changing. He then points out that at each instant the arrow simply is where it is, just as it would be if it were at rest. Hence he concludes that there can be no such thing as a *state* of motion, and therefore, adhering to the view that a state of motion is essential to motion, he infers that there can be no motion and that the arrow is always at rest.

Zeno's argument, therefore, though it does not touch the mathematical account of change, does, *prima facie*, refute a view of change which is not unlike M. Bergson's. How, then, does M. Bergson meet Zeno's argument? He meets it by denying that the arrow is ever anywhere. After stating Zeno's argument, he replies: "Yes, if we suppose that the arrow can ever *be* in a point of its course. Yes again, if the arrow, which is moving, ever coincides with a position, which is motionless. But the arrow never *is* in any point of its course" (*C. E.*., p. 325). This reply to Zeno, or a closely similar one concerning Achilles and the Tortoise, occurs in all his three books. Bergson's view, plainly, is paradoxical; whether it is *possible*, is a question which demands a discussion of his view of duration. His only argument in its favor is the statement that the mathematical view of change "implies the absurd proposition that movement is made of immobilities" (*C. E.*, p.

325). But the apparent absurdity of this view is merely due to the verbal form in which he has stated it, and vanishes as soon as we realize that motion implies relations. A friendship, for example, is made out of people who are friends, but not out of friendships; a genealogy is made out of men, but not out of genealogies. So a motion is made out of what is moving, but not out of motions. It expresses the fact that a thing may be in different places at different times, and that the places may still be different however near together the times may be. Bergson's argument against the mathematical view of motion, therefore, reduces itself, in the last analysis, to a mere play upon words. And with this conclusion we may pass on to a criticism of his theory of duration.

Bergson's theory of duration is bound up with his theory of memory. According to this theory, things remembered survive in memory, and thus interpenetrate present things: past and present are not mutually external, but are mingled in the unity of consciousness. Action, he says, is what constitutes being; but mathematical time is a mere passive receptacle, which does nothing and therefore is nothing (*C. E.*, p. 41). The past, he says, is that which acts no longer, and the present is that which is acting (*M. and M.*, p. 74). But in this statement, as indeed throughout his account of duration, Bergson is unconsciously assuming the ordinary mathematical time; without this, his statements are unmeaning. What is meant by saying "the past is essentially *that which acts no longer*" (his italics), except that the past is that of which the action is past? The words "no longer" are words expressive of the past; to a person who did not have the ordinary notion of the past as something outside the present, these words would have no meaning. Thus his definition is circular. What he says is, in effect, "the past is that of which the action is in the past." As a definition, this cannot be regarded as a happy effort.

And the same applies to the present. The present, we are told, is "*that which is acting*" (his italics).[3] But the word "is" introduces just that idea of the present which was to be defined. The present is that which *is* acting as opposed to that which *was* acting or *will be* acting. That is to say, the present is that whose action is in the present, not in the past or in the future. Again the definition is circular. An earlier passage on the same page will illustrate the fallacy further. "That which constitutes our pure perception," he says, "is our dawning action. . . . The *actuality* of our perception thus lies in its *activity,* in the movements which prolong it, and not in its greater intensity: the past is only idea, the present is ideo-motor" (*ib.*). This passage makes it quite clear that, when Bergson speaks of the past, he does not mean the past, but our present memory of the past. The past when it existed was just as active as the present is now; if Bergson's account were correct, the present moment ought to be the only one in the whole history of the world containing any activity.

In earlier times there were other perceptions, just as active, just as actual in their day, as our present perception; the past, in its day, was by no means only idea, but was in its intrinsic character just what the present is now. This real past, however, Bergson simply forgets; what he speaks of is the present idea of the past. The real past does not mingle with the present. Our memory of the past does of course mingle with the present, since it is part of it; but that is a very different thing.

The whole of Bergson's theory of duration and time rests throughout on the elementary confusion between the present occurrence of a recollection and the past occurrence which is recollected. But for the fact that time is so famil-

[3] Similarly in *Matter and Memory* (p. 193) he says it is a question whether the past has ceased to exist, or has only *ceased* to be useful. The present, he says, is not that which is, but that which *is* being made. The words I have italicized here really involve the usual view of time.

iar to us, the vicious circle involved in his attempt to deduce the past as what is no longer active would be obvious at once. As it is, what Bergson gives is an account of the difference between perception and recollection—both *present* facts—and what he believes himself to have given is an account of the difference between the present and the past. As soon as this confusion is realized, his theory of time is seen to be simply a theory which omits time altogether.

The confusion between present remembering and the past event remembered, which seems to be at the bottom of Bergson's theory of time, is an instance of a more general confusion which, if I am not mistaken, vitiates a great deal of his thought, and indeed a great deal of the thought of most modern philosophers—I mean the confusion between an act of knowing and that which is known. In memory, the act of knowing is in the present, whereas what is known is in the past; thus by confusing them the distinction between past and present is blurred. In perception, the act of knowing is mental, whereas what is known is (at least in one sense) physical or material; thus by confusing the two, the distinction between mind and matter is blurred. This enables Bergson to say, as we saw, that "pure perception, which is the lowest degree of mind. . . . is really part of matter." The act of perceiving is mind, while that which is perceived is (in one sense) matter; thus when these two are confused, the above statement becomes intelligible.

Throughout *Matter and Memory,* this confusion between the act of knowing and the object known is indispensable. It is enshrined in the use of the word "image," which is explained at the very beginning of the book.[4]

[4] Bergson's use of the word "image" is made clearer by a very penetrating analysis of Berkeley in a recent article, "L'Intuition Philosophique" (*Revue de Métaphysique et de Morale,* Nov. 1911). This article displays very distinctly the profound influence of Berkeley on Bergson's thought. Bergson's "image" is practically Berkeley's "idea."

He there states that, apart from philosophical theories, everything that we know consists of "images," which indeed constitute the whole universe. He says: "I call *matter* the aggregate of images, and *perception of matter* these same images referred to the eventual action of one particular image, my body" (*M. and M.*, p. 8). It will be observed that matter and the perception of matter, according to him, consist of the very same things. The brain, he says, is like the rest of the material universe, and is therefore an image if the universe is an image (p. 9).

Since the brain, which nobody sees, is not, in the ordinary sense, an image, we are not surprised at his saying that an image can *be* without *being perceived* (p. 27); but he explains later on that, as regards images, the difference between *being* and *being consciously perceived* is only one of degree (p. 30). This is perhaps explained by another passage in which he says: "What can be a non-perceived material object, an image not imaged, unless it is a kind of unconscious mental state?" (p. 183). Finally (p. 304) he says: "That every reality has a kinship, an analogy, in short a relation with consciousness—this is what we concede to idealism by the very fact that we term things 'images.'" Nevertheless he attempts to allay our initial doubt by saying that he is beginning at a point before any of the assumptions of philosophers have been introduced. "We will assume," he says, "for the moment that we know nothing of theories of matter and theories of spirit, nothing of the discussions as to the reality or ideality of the external world. Here I am in the presence of images" (p. 1). And in the new Introduction which he wrote for the English edition he says: "By 'image' we mean a certain existence which is more than that which the idealist calls a *representation,* but less than that which the realist calls a *thing,*—an existence placed halfway between the 'thing' and the 'representation'" (p. vii).

The distinction which Bergson has in mind in the above is not, I think, the distinction between the imaging as a mental occurrence and the thing imaged as an object. He is thinking of the distinction between the thing as it is and the thing as it appears, neither of which belongs to the subject. The distinction between subject and object, between the mind which thinks and remembers and has images on the one hand, and the objects thought about, remembered, or imaged—this distinction, so far as I can see, is wholly absent from his philosophy. Its absence is his real debt to idealism; and a very unfortunate debt it is. In the case of "images," as we have just seen, it enables him first to speak of images as neutral between mind and matter, then to assert that the brain is an image in spite of the fact that it has never been imaged, then to suggest that matter and the perception of matter are the same thing, but that a non-perceived image (such as the brain) is an unconscious mental state; while finally, the use of the word "image," though involving no metaphysical theories whatever, nevertheless implies that every reality has "a kinship, an analogy, in short a relation" with consciousness.

All these confusions are due to the initial confusion of subject and object. The subject—a thought or an image or a memory—is a present fact in me; the object may be the law of gravitation or my friend Jones or the old Campanile of Venice. The subject is mental and is here and now. Therefore, if subject and object are one, the object is mental and is here and now; my friend Jones, though he believes himself to be in South America and to exist on his own account, is really in my head and exists in virtue of my thinking about him; St. Mark's Campanile, in spite of its great size and the fact that it ceased to exist ten years ago, still exists, and is to be found complete inside me. These statements are no travesty of Bergson's theories of

space and time; they are merely an attempt to show what is the actual concrete meaning of those theories.

The confusion of subject and object is not peculiar to Bergson, but is common to many idealists and many materialists. Many idealists say that the object is really the subject, and many materialists say that the subject is really the object. They agree in thinking these two statements very different, while yet holding that subject and object are not different. In this respect, we may admit, Bergson has merit, for he is as ready impartially to identify subject with object as to identify object with subject. As soon as this identification is rejected, his whole system collapses: first his theories of space and time, then his belief in real contingency, then his condemnation of intellect, then his account of the relations of mind and matter, and last of all his whole view that the universe contains no things, but only actions, movements, changes, from nothing to nothing, in an endless alternation of up and down.

Of course a large part of Bergson's philosophy, probably the part to which most of its popularity is due, does not depend upon argument, and cannot be upset by argument. His imaginative picture of the world, regarded as a poetic effort, is in the main not capable of either proof or disproof. Shakespeare says life's but a walking shadow, Shelley says it is like a dome of many-colored glass, Bergson says it is a shell which bursts into parts that are again shells. If you like Bergson's image better, it is just as legitimate.

The good which Bergson hopes to see realized in the world is action for the sake of action. All pure contemplation he calls "dreaming," and condemns by a whole series of uncomplimentary epithets: static, Platonic, mathematical, logical, intellectual. Those who desire some prevision of the end which action is to achieve are told that an end foreseen would be nothing new, because desire, like mem-

ory, is identified with its object. Thus we are condemned, in action, to be the blind slaves of instinct: the life-force pushes us on from behind, restlessly and unceasingly. There is no room in this philosophy for the moment of contemplative insight when, rising above the animal life, we become conscious of the greater ends that redeem man from the life of the brutes. Those to whom activity without purpose seems a sufficient good will find in Bergson's books a pleasing picture of the universe. But those to whom action, if it is to be of any value, must be inspired by some vision, by some imaginative foreshadowing of a world less painful, less unjust, less full of strife than the world of our every-day life, those, in a word, whose action is built on contemplation, will find in this philosophy nothing of what they seek, and will not regret that there is no reason to think it true.

B. RUSSELL.

CAMBRIDGE, ENGLAND.

# PSYCHOTHERAPIC CULTS:[1]

## CHRISTIAN SCIENCE; MIND CURE; NEW THOUGHT.

THE most noteworthy religious event since the Reformation is perhaps the appearance in the United States of a number of religious movements which may be grouped together under the designation of psychotherapic cults. The foremost of them is "Christian Science," founded by Mrs. Mary Baker Eddy.

I hasten to add that the value of these cults does not, in my mind, belong to their "metaphysics," considered as a philosophical system. It is the product of ignorant and ill-trained minds. Much of it defies logic and offends common sense. But the defects which in the eyes of many wholly damn these movements might conceivably be removed, and there would remain important elements of a new religious faith acceptable to the modern world.

I shall try to show that the psychotherapic movements in their essential teaching are popularized and distorted formulations, on the one hand, of important truths regarding the "power of thought" over body to which psychology has recently given added significance, and, on the other, of a non-theistic philosophy allied to the absolute idealism of modern metaphysics. Although they distort contemporary thought, they do not intend to oppose it. They wish rather to build upon it.

[1] A discussion of other contemporary movements will be found in the author's book, *A Psychological Study of Religion: Its Origin, Function and Future*, Macmillan, 1912.

These new cults are forcible reminders of the fact that belief in a saving power is a condition of the existence of religion, and also that the desire for deliverance from moral and physical miseries and for the realization of ideals continues to be the motive of religious life, just as it was in the days of Gautama the Enlightener, and of Jesus the Healer.

* * *

The mind-cure books announce "the discovery of the might of truth in the treatment of disease as well as of sin," "the vital law of true life, true greatness, power, and happiness." They claim to be "systems of transcendental medicine," or of "psychic therapeutics." They purpose to minister to those who "would exchange impotence for power, weakness and suffering for health and strength, pain and unrest for peace, poverty for fulness and plenty." They proclaim "the birthright of every man born into the world to be physically whole and mentally happy." Their claims have an extravagant sound, but no more so than those made for "faith" by the New Testament writers who declared it would remove mountains and secure eternal blessedness after death. Nothing but vital experiences could have inspired the enthusiasm and the assurance with which these modern zealots proclaim the abounding efficacy of their "truth."

If they call themselves Christians, it is not in the traditional sense. Of traditional Christianity they speak respectfully, but they want a new dogmatics. They say, "The time for thinkers has come. Truth, independent of doctrines and time-honored systems, knocks at the portal of humanity."[2] In another of their aggressive little books one reads: "Unrest is universal. The old landmarks are disappearing. . . . Creed and dogma are things of the past;

[2] Mary G. Baker Eddy, *Science and Health*, 1908, Preface.

religious ceremonial and form no longer interest the masses."[3]

The impression these cults have produced on thoughtful religious people is well expressed in this passage:

"Renan with his usual intuition declared that if it [the religion of the future] were already in our midst, few of us would know it.

"The prediction has proved true. The new religious movement Christian Science has spoken a language so foreign to cultivated ears, its interpretation of the Bible is so false, it is so obviously committed to errors, illusions, and aberrations of every sort, that the intelligent have been disposed to shrug their shoulders in contempt and to ignore it. And yet they have not been able to ignore it altogether. Every once in a while this curious superstition proves its existence with unexpected power. We see a hard-headed business man totally devoid of religious sentiment undergo a new kind of conversion which leaves him as devout and ardent as a Christian of the first century. An ailing wife or daughter whom no physician has been able to help, through some mysterious means is restored to health and happiness. The victim of an enslaving habit, apparently with very little effort and without physical means, sufferings, or relapse, finds himself free. We enter a home where the new belief reigns and we find there a peace to which we are strangers.

"All over the country solid and enduring temples are reared by grateful hands and consecrated to the ideal and name of Mrs. Eddy. And this strange phenomenon has occurred in the full light of day, at the end of the nineteenth and at the beginning of the twentieth century, and these extraordinary doctrines have propagated themselves not in obscure corners of the earth, among an illiterate and fanatical population, but in the chief centers of American

[3] Charles B. Patterson, *A New Heaven and a New Earth*, Preface.

civilization. Such facts may well cause the philosophical student of religion to reflect.[4]

In these movements is restored the alliance between the art of healing the body and the art of healing the soul, which was always a leading characteristic of the higher religions during their period of greatest vitality. To the masses the most impressive aspect of religions has always been their power to heal the body. It was so in the early ministry of Christ and during the first Christian centuries. It is so now with these psychotherapists. And this revival acquires great significance from the fact that it can now be grounded upon the deeper understanding of the interrelation of mind and body, which we owe to modern science.

Speaking of the "four noble truths" of Buddhism, (Satyāni), i. e., the four axioms or certainties: the existence of suffering, the origin of suffering, the emancipation from suffering and the path that leads to the emancipation from suffering, Kern says: "It is not difficult to see that these four Satyas are nothing else but the four cardinal articles of Indian medical science, applied to the spiritual healing of mankind, exactly as in the Yoga doctrine. This connection of the Aryasatyas with medical science was apparently not unknown to the Buddhists themselves." And concerning the twelvefold causal root of the evil of the world, the twelve Nidānas (causes), he declares that they stand to the four Satyas 'in the same relation as pathology to the whole system of medical science.' Now the four truths and the twelve causes are fundamental facts upon which Gautama's scheme of deliverance is built."[5]

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My chief effort will be to get from the writings of the leaders of these therapeutic schools a clear idea of the

[4] Elwood Worcester, Samuel McComb, Isador H. Coriat, *Religion and Medicine*, New York, 1908, pp. 8-10.

[5] Kern, *Manual of Buddhism*, Grundriss der indo-arischen Philologie und Altertumskunde, Vol. III, No. 8, pp. 46-47.

power with which they expect to regenerate humanity, and then to consider its adequacy. Whatever their affiliations, these writers practically agree on the points that most interest us. I do not shrink from putting before my readers, to begin with, brief quotations from two of the most extravagant and crude of these authors; for even they find followers among people who prove themselves intelligent and sensible in the affairs of life.

T. Troward, a leader of Mental Science (not a disciple of Mrs. Eddy), late divisional judge in Punjab and Edinburgh Lecturer on Mental Science, teaches the existence of an unlimited, impersonal, though intelligent power, which man may press into service, or appropriate to himself. His view of man's relation to that power is curious. The individual can call it into action and give it direction, "because it is in itself impersonal though intelligent." "It will receive the impress of his personality, and can therefore make its influence felt far beyond the limits which bound the individual's objective perception of the circumstances with which he has to deal. It is for this reason that I lay so much stress on the combination of two apparent opposites in the Universal Mind, the union of intelligence with impersonality....How do we know what the intention of the Universal Mind may be? Here comes in the element of impersonality. It has *no intention,* because it is *impersonal*..... Combining, then, these two aspects of the Universal Mind,....we find precisely the sort of natural force we are in want of, something which will undertake whatever we put into its hands without asking questions or bargaining for terms, and which, having undertaken our business, will bring to bear on it an intelligence to which the united knowledge of the whole human race is as nothing, and a power equal to this intelligence."[6]

[6] T. Troward, *The Edinburgh Lectures on Mental Science,* The Arcane Book Concern, 1909, Chicago, pp. 66-68.

I find it difficult to conceive an unlimited impersonal intelligence which has no intention and which individual intelligence may direct. But in fairness to the abstruse judge, I must add that this difficulty is no greater than that presented by Hegel's conception of the Absolute Mind.

In the work of W. F. Evans we meet a consistent pantheism. He strives to give to his opinions an impressive background compounded of modern science, antique pantheism, and modern idealism. How vast and accurate is his knowledge will appear in the following passage. I quote it without apology as another instance of a type of conception apparently rational enough to be accepted by many intelligent people. "The soul of man is a part, so to speak, of the *anima mundi,* the soul of the world." The power of the healing thought "issues from the spiritual world of which our minds are a part, for all ideas belong to that boundless realm of life." "It is stored up in exhaustless and overflowing abundance in the bosom of nature....it can be controlled in its lower degrees of manifestation by the intelligent will of man, which is the highest form of its development and expression." "This grand whole....the universal world of spiritual intelligence is called in Sanskrit, *Addi-Budda.* In the writings of Paul it is called the Christ....It is identical with what is called magnetism, and is also that which the philosophers have called the divine *nous.*"[7]

One of the ablest and sanest writers of New Thought, Ralph Waldo Trine, in a book which has passed its seventy-fifth thousand, also announces a pantheistic gospel of an infinite power at the service of man. "The great central fact of the universe is that spirit of Infinite Life and Power that is back of all, that animates all, that manifests itself in and through all; that self-existent principle of life from

[7] W. F. Evans, *The Primitive Mind-Cure: Elementary Lessons in Christian Philosophy and Transcendental Medicine.*

which all has come, and not only from which all has come, but from which all is continually coming."

"This Infinite Power is creating, working, ruling through the agency of great immutable laws and forces that run through all the universe, that surround us on every side. Every act of our every-day lives is governed by these same great laws and forces."

"In a sense there is nothing in all the great universe but law." But the presence of laws indicates a force back of them. "This Spirit of Infinite Life and Power that is back of all is what I call God."

"God, then, is this Infinite Spirit which fills all the universe with Himself alone, so that all is from Him and in Him, and there is nothing that is outside. . . . He is. . . . our very life itself." "In essence the life of God and the life of man are identically the same, and so are one. They differ not in essence, in quality; they differ in degree."

". . . . if the God-powers are without limit, does it not then follow that the only limitations man has are the limitations he sets to himself, by virtue of not knowing himself?"

*"The great central fact in human life, in your life and in mine, is the coming into a conscious, vital realization of our oneness with this Infinite Life, and the opening of ourselves to this divine overflow."* This means simply "that we are recognizing our true identity, that we are bringing our lives into harmony with the same great laws and forces, and so opening ourselves to the same great inspirations as have all the prophets, seers, sages, and saviours in the world's history, all men of truly great and mighty power."[8] He does not hesitate to use the term "God-man."

[8] Ralph Waldo Trine, *In Tune with the Infinite or Fullness of Peace, Power, and Plenty*, Thomas Y. Crowell and Co., New York, pp. 11-20.

*Christian Science.*

It seems almost incredible that one professing to be a Christian should teach the impersonality of the divine nature. And yet this is undoubtedly what Mrs. Eddy does, and in this respect she agrees with those from whom I have just quoted. The term that she prefers as a name for the Divine Power is Principle. As synonyms she uses Life, Truth, Love, God. In the earlier editions of *Science and Health,* it is written that God "is not a person, God is Principle."[9] This is undoubtedly the standpoint of her later writings also. But in them, probably because of the pressure of adverse public opinion, she insists less than at the beginning of her career upon the impersonality of Principle, and the word "person" appears more frequently. "Once in 1898, Mrs. Eddy hints that God may be personal 'if the term personality, as applied to God, means infinite personality,' and Mr. Farlow in 1907 assures the Rev. Edgar P. Hill that Mrs. Eddy does believe that 'God is person in the infinite sense.' "[10] I take the following passages from the same book: "Principle in her theology gathers up into itself all the concepts we habitually associate with God, except the most important—personality. Before her book appeared in 1875, she was telling her pupils, as two of them informed me, that they could make no progress till they had banished from their minds the thought of God as a person. She instructed Richard Kennedy 'to lay special stress' in healing patients on the impersonality of God. This is the commanding thought that rings through the first chapter of the first edition of *Science and Health.*"

"Mrs. Eddy's pantheism is unnecessary, and yet its origin was inevitable in a mind as literal as hers. Quimby often spoke of God as Principle. In the Quimby manu-

[9] Mary G. Baker Eddy, *op. cit.,* 3d ed., 1881, I, 67; II, 27.

[10] Lyman P. Powell, *Christian Science, the Faith and its Founder,* pp. 139-140.

script from which, for several years, Mrs. Eddy taught, no sentence is more startling than the sentence 'God is Principle.' "

"For more than thirty years Mrs. Eddy has been solemnly asserting that in 1866 she received a 'final revelation.' Now this 'final revelation,' which was finally as well as first expressed in 1875, in *Science and Health,* is saturated with thought that God is not a person. In the very first chapter we are informed that 'God is Principle, not person,' [I do not find that expression in the first chapter of the 1908 edition, but it is in *No and Yes,* published in 1909] that Jesus preached the impersonality of God, that the error of believing in the personality of God crucified Jesus, that the trouble with conventional Christianity to-day is that it makes God a person....' (Pages 137-140).

On the other hand, in the seventy-third edition of *No and Yes,* published in 1909, a pamphlet intended "to correct involuntary as well as voluntary error," we read: "Is there a personal Deity? God is Infinite. He is neither a limited mind nor a limited body. God is Love; and Love is Principle, not person. What the person of the Infinite is, we know not; but we are gratefully and lovingly conscious of the fatherliness of this Supreme Being. God is individual, and man is his individualized idea.... Limitless personality is inconceivable....Of God as person, human reason, imagination and revelation give us no knowledge.

"When the term divine Principle is used to signify Deity it may seem distant and cold, until better apprehended. This Principle is Mind, Substance, Life, Truth, Love. When understood, Principle is found to be the only term that fully conveys the ideas of God,—one Mind, a perfect Man, and divine Science."[11] This Principle, though not a person, "is intelligence."

Although she wrote, "God is All in all," and "All in all

[11] Eddy, *No and Yes,* 1909, pp. 19, 20.

is God,"[12] she will not be called a pantheist. In the edition of *No and Yes* already quoted, she claims that "Christian Science refutes pantheism, finds Spirit neither in matter nor in the modes of mortal mind. It shows that matter and mortal mind have neither origin nor existence in the eternal Mind. . . . For God to know, is to be; that is, what He knows must truly and eternally exist. If He knows matter, and Matter cannot exist in Mind, then mortality and discord must be eternal."[13]

Her pantheism is in any case not materialistic, since she holds matter to be unreal, a deception of mortal mind. Hers is an idealistic pantheism, such as an ignorant person of a thoroughly optimistic temperament might evolve on the basis of imperfect knowledge of absolute idealism and from observation of the mastery of mind over body.

The writings of Mrs. Eddy's disciples reflect the uncritical, pantheistic idealism of their leader. Their favorite phrases are such as these: "God's presence is the presence of love;" "God is life everywhere present;" "One life fills all, it is the Perfect Life."

The similarity of the essential aspects of New Thought and Christian Science to the mystical element in Christianity is evident. Both give clear expression to the anti-isolation motive, to a dynamic belief in oneness-with-the-whole, and both feel the essence of the cosmic plasma to be love. Man is steeped in all-embracing Love. He need only place himself in unison with the everlasting, all-comprehending life-force and the fulness of life will be his. How love can be an attribute of an impersonal power does not seem to give Mrs. Eddy one moment of uneasiness.

In their curative practices, the psychotherapic cults have the benefit of the recent discoveries concerning the effects of suggestion. Regarding their methods, I may

[12] Eddy, *Science and Health*, 1898, p. 7.

[13] Eddy, *No and Yes*, pp. 15, 16.

say here merely that they tend to place the person, as do the practices of the other ethical religions, in a state of increased suggestibility, a state described in part by the words relaxedness, collectedness, monoideism, meditation, communion. This condition of the subject aids greatly in the realization of the expected benefits. The efficacy of these curative methods is sufficiently demonstrated by the wonderful extension of the movements. In every walk of life people bear witness to the saving grace that is in Christian Science or in New Thought. The forces of a new life have welled up within them; the burdens of existence have lightened, nay, have disappeared; and now they walk through life contented, hopeful, and aggressively benevolent.

The following is an example of what people find in Christian Science apart from the cure of disease:

"I accepted *Science and Health* without expecting it to offer more than a human theory about life,—even the name did not lead me to expect it to be religious; in fact, the chief incentive to my reading it at that time was the great kindness and sincere sympathy evinced by my friend, who placed a copy at my disposal.....I started timidly at first, and prayerfully, lest it should be misleading, but before I had gone very far I experienced that wonderful spiritual quickening which is so often spoken of in our meetings. I wish I could tell exactly what the experience meant to me, the wonderful awakening I had; how old things vanished and all things became new. It seemed as if the burdens, perplexities, doubts, and fears had all suddenly rolled away; as if the sun had emerged from behind the clouds, and everything was again bright and beautiful.

"And what a feeling of strength, hope, and courage came! Those old troublesome questions, especially the question of death, were explained, and I felt a wonderful release to know that death was not of God. I read and

reread the latter part of the chapter on Christian Science Practice, where that glorious truth is explained; it was so beautiful, so natural, and so true. There was such perfect joy to me in that freedom, that I used to declare over and over again, of those who had just passed from us (the members of our home circle), 'They are not dead;' and so free was I made from the old bondage, that never since then has the thought of that change affected me as it did before."[14]

Unnecessary importance is attached by the critical public to the vagaries of Christian Science and of New Thought; for instance, to the denial of the reality of matter, and therefore of disease; to the wild hopes of some of their prophets that "the time will certainly come when the highly developed man will have the power to lay down or take up his life through a conscious knowlege of the laws of eternal being and the direct application of these laws to his own life."

When I say "wild hopes," I speak as the prosaic man that I am. No less a philosopher than Bergson has expressed that same hope of overcoming death.

An apologist of the psychotherapic sects would be justified in making the following claims:

1. The salvation they promise is first of all for *this life*.

2. The soul is not saved independently of the body. The nefarious asceticism of older faiths is impossible on the principles of Christian Science.

3. Their ideal involves efficiency in the conduct of this life.

4. Their conception of salvation is free from anything miraculous. They dispense with the wonders of the Fall,

[14] *Christian Science Sentinel*, Dec. 3, 1901.

[15] Charles B. Patterson, *op. cit.*, Preface.

of the self-sacrifice of a divine personage, and of salvation by his atonement.

5. They divert attention from the sense of guilt and suffering, and direct it to an immediately accessible healing and invigorating power.

6. Although they usually define the aim of life in terms of power, happiness, and love, they cannot fairly be charged either with insensitiveness to moral values, or with indifference to the ethical advancement of mankind.

7. Despite its extravagance, their "metaphysics" may be regarded as a formulation, crude and distorted, of a *Weltanschauung* made unavoidable by modern knowledge, —a *Weltanschauung*, opposed in several important respects to the traditional but no longer acceptable Christian philosophy.

8. These cults have proved their value by their results.

In estimating the chances of continued life of religious movements, one should bear in mind that vitally beneficial beliefs may carry a heavy load of error and even of absurdity. The Christian religion was not destroyed by the expectation of the second coming of the Lord and of the end of the world, by extravagant notions of the power of faith, by absurd or incomprehensible doctrines regarding the means of salvation, the resurrection of the body, and the like. There is enough substantial, practical truth in Christianity to bear the enormous doctrinal dead weight it carries even to this day. It may be possible for the psychotherapic doctrines to be purified in a reformation which would either remove entirely or drive into side-currents most of the offensive tenets.

J. H. Leuba.

Bryn Mawr College, Pa.

# THE MYSTERY OF LIFE.

## A POETIZATION OF "THE HAKO"— A PAWNEE CEREMONY.

BY HARTLEY BURR ALEXANDER.

PREFATORY NOTE. The 22d Annual Report of the Bureau of American Ethnology contains Alice C. Fletcher's record of "The Hako: a Pawnee Ceremony." This record is the foundation of the present work. Miss Fletcher, in her rhythmic renderings of the Indian songs, has incorporated meanings given in the explanations of the leader of the ceremony as well as the literal sense of the Indian texts; the result being a series of admirable translations, abounding in telling phrases. The version here presented has drawn freely upon Miss Fletcher's fine renderings; but as "The Mystery" was designed to emphasize the universal elements in the Indian thought, it necessarily involved generalization and amplification of the primitive expression, as well as rearrangement of materials.—The piece was conceived as a dramatic pageant, with musical accompaniment, as will appear from its form.

### DESCRIPTION OF THE SYMBOLS.

*The Persons:*

The LEADER, a Priest. He wears leggings and moccasins, and a robe girt about his body, leaving shoulders and arms bare; in his hair is a feather of white eagle's down; he carries the brown-plumed wand.

Five ACOLYTES, dressed like the Leader. They carry the ceremonial articles and act as assistants to the Leader.

The CHORUS, consisting of

(A) The SEMI-CHORUS OF FATHERS, led by the CHIEF OF THE FATHERS. They are dressed in leggings and moccasins and ceremonial shirts, ornamented with blue and white. They wear bonnets of white eagle's plumes. The Chief carries a calumet and his bonnet extends in streamers of plumes down his back. In Part II the bonnets are left off, and all wear blankets, symbolic of night.

(B) The SEMI-CHORUS OF THE CHILDREN, led by the CHIEF OF THE CHILDREN. They are dressed like the Fathers, except that their colors are green and red and their bonnets adorned with brown plumes. In Part II they also leave off the bonnets and wear blankets.

The CHILD.

The PERSONATOR OF THE MORNING-STAR, dressed in red, and wearing a red plume; spread wings are attached to his wrists.

*The Powers*:

The BLUE SKY, abode of the FATHER OF HEAVEN, the Mighty Power.

The POWERS OF HEAVEN: The MORNING STAR, Herald of Day; the DAWN, Child of Heaven and of Night; the SUN, Father of Day and of Life; the Four WINDS from the Four Quarters of the World, where are the Paths from Earth to Heaven.

MOTHER EARTH, whose Child is the Green Vegetation symbolized by the CORN SPIRIT, and who sustains life with the running Waters which are the WATERS OF LIFE and symbolize the continuing generations of Mankind.

The EAGLE, Chief of the Birds who are the Mediators between the Mighty Power and Man; Conductor of the Visions, dwelling in the lower Heaven, down to Man; Symbol of the care which the Father of Heaven has for his Children,—the brown plumes being emblematic of the Female Eagle in her care for her nestlings, the white plumes of the protecting Male Eagle: the place of the white is always outermost.

*Emblems and Ceremonial Articles*:

The BROWN-PLUMED WAND, borne by the Leader: a hollow stem, painted blue, emblematic of the Sky, and adorned with a fan of the brown plumes of the Female Eagle; also, with a Duck's head and breast, one end of the stem being thrust through the mandibles; with a tuft of Owl feathers; with red and white streamers, emblematic of Sun and Stars.

The WHITE-PLUMED WAND, borne by an Acolyte: like the preceding except that the stem is green, symbolic of Earth, and the plumes are the white plumes of the Male Eagle.

The SPREAD WINGS OF AN EAGLE, mounted like the wings on the caduceus of Mercury, except that each wing is on a detachable staff so that they can be held separately, simulating flight, or conjoined, forming a banner.

The CORN, a light sheaf of maize with unhusked ears, symbolic of the Corn Spirit and of the Vegetation which is the offspring of life-giving Mother Earth.

The BOWL, hewn from the living wood, a part of Earth's green covering, and painted blue as symbolizing the blue Sky. In it is borne water from a running stream, symbolic of the vigor and strength which Earth gives in the Waters of Life and of the continuance of life in the on-flowing generations of men.

A tray with implements for Fire-making; a tray with ceremonial Tobacco; a tray with four cups, one containing red, one blue, and one green paint, and one with oil and fat. Trays of bread; jugs of water; turfs for building the hearth-altar.

*The Place*:

The PLACE is a sward of level green on the open prairie. Above is the blue sky with a few fleecy clouds. At the Western side of the sward, forming the background, is a hedge of greenery, with three openings or gates.

The Northern Gate is the Gate of the Fathers,—North is the side of Night and of the need of protection.

The Southern Gate is the Gate of the Children,—the South is the Winter home of the birds, the side of peace and of plenty.

The Middle Gate, facing the Place of Sunrise, is the Gate of the Leader and Acolytes. It opens from the Holy Place.

*Color Symbolism*:

BLUE symbolizes the abode of the Powers Above and of the Father of Heaven; GREEN symbolizes the Earth and life-giving food; RED is the color of Life, of the life-blood and of the Morning Star who is herald of light and life; WHITE symbolizes Sunlight, the fleecy Clouds and the Winds, and hence the breath of Heaven, the Breath of Life.

## PART I. THE COMING OF THE CORN.

### THEME I.

*Orchestral Prelude. Enter from the Central Gate the Leader and Acolytes; from the North Gate, the Fathers; from the South Gate, the Children. The Acolytes and the Chorus remain at the Rear; the Leader advances to the Forefront.*

*The Leader intones*:

I.

Give heed! Give heed!
Give heed, O ye People!
Unto the Abode of Life give ye heed,
And unto the Powers thereof
Let your hearts be turned in reverence....

II.

Lift up your gaze!
Unto the blue and doming Skies
Lift up your gaze,—
Where dwelleth the Father of Heaven,
Where dwelleth the Father of Life,
Yea, from everlasting to everlasting.
Lift up your gaze
Unto the Father!....

In the Circle of the Heavens He hath set
The manifestations of His glory,—
The bright and shining Sun,
Which giveth forth the Light of Day
And answereth the hymn wherewith His creatures
Waken at Morn,—
In the Circle of the Heavens He hath established the Sun
To be a sign of His presence by Day,
And the quiet Stars hath He set to be His nightly ministers..

The Four Winds
From the Four Pathways of the Skies,—
East, South, North, West,—
Breathe forth His Word and His Life
Throughout the Lodge of Heaven:
Yea, the music of His Word
And the gladness of Life
Breathe they forth
Through the Four Quarters of the World....

Lift up your gaze
Unto the blue and doming Skies!....

III.

Upon the Earth
Let your thoughts descend,—
Our Mother Earth!
From her dark and fruitful womb ye are sprung,
And at her nourishing bosom ye are fed:
She is the Great Mother
Who keepeth us in life
And at death receiveth us:
Think on the Mother!

Her garment is the fair and flowing green,
The verdure of the hills is her habiliment,
Whence they that move obtain their strength
And the Sons of Men their sustenance:
Who is the Giver of Food unto her children.

As milk streameth from the breast,
From her ancient hills
And the cool depths of her yearly snows
The clear and living Waters are poured forth,
To be for her children their drink and their refreshing:
Yea, unto them that thirst She giveth the Waters of Life.
Think on the Mother!....

IV.

Upon the Earth
Let your thoughts descend in reverent heed:
Let them be lifted up
To the blue and doming Skies!
Upon Earth and upon Heaven let your thoughts be placed,
For they are the Abode of Life and of the Powers thereof...

THEME II.

*Roll of drums. The Chorus advances a pace, crying in unison:*

Look down! We gaze afar on your dwelling!
Ye Mighty Ones, look down!

*During the orchestral development that follows, the Semi-Chorus of Fathers advances to the center of the sward where they form a circle, with an opening to the East and one to the West, thus simulating the circular outline of the walls of an earth lodge. With their hands they indicate the building of walls. During this action they chant:*

Ye of the Winds, behold us!
Ye Thunder gods, behold us!
Wielders of Leven, behold us!
Bringers of Death, behold us!

Ye of the Rains, behold us!
Ye of the Clouds and the Soil!
Givers of Increase, behold us!
Givers of Life, behold us!

We establish here a dwelling,—
A Wall of Defense,
A House of Life,
A Place that is Holy!

*Full Chorus*:

Look down! We gaze afar on your dwelling!
Ye Mighty Ones, look down!

*Semi-Chorus of Children advances, from the Western opening, within the circle—the symbolic lodge—formed by the Fathers. They carry turfs which they build into an hearth-altar at the center of the circle as they file past. They form into two half circles, North and South, within the circle of the Fathers. During the action they chant*:

Spirits of Heaven, behold us!
Spirits of Earth, behold us!
Ye Shining Ones, behold us!
Ye Darkling Ones, behold us!
Ye that measure out the ways of men....

Here we build unto you an Altar,
Whereof the flame is the prayer of man
Ascending....

*The Leader enters the encircled space from the Eastern opening, three Acolytes bearing fire-making implements enter from the West. At the Altar the Leader lays a fire and sends up a pillar of smoke, like an Indian signal smoke. The music is the music of fire and of prayer. As the smoke ascends—*

*The Chorus*:

See! The Pillar of Smoke ascendeth
Up to the dome of Heaven
Where God abideth....

*The Leader*:

As riseth the smoke of the Altar,
So the spirit of man upstriveth,
So the cry of his heart upmounteth,
Unto the deeps of the Blue,
Unto the Silence of God....

*The Chorus*:

Speed aloft!
Bearing our supplication,
Bearing our prayer!

THEME III.

*The flutes strike in with the clear piercing music of the Eagle. The Leader gazes into the Eastern Sky. He raises his arm impressively, crying:*

Lo, where cometh His answer—
The Eagle of the Chief of Heaven!

*The Chorus circles North and South, bringing their faces to the East, and then, during continuous circling motion:*

Behold, an Eagle now is circling, widely circling above us!

*Semi-Chorus of Children, circling to the South:*

As the mother-bird circleth her nestlings, careful for her chicks,
She circleth us, hovering....

*Semi-Chorus of Fathers, circling to the North:*

She is the Eagle of God!
Of Him who is Father of Heaven,
Who ruleth the quartered Earth,
And sendeth His Will by the Eagle
Over the windy Pathways
That lead from Man up to God....

*Semi-Chorus of Children:*

She is the Eagle of God!
The sign that He hath sent us
That we are in His eyes
As to the mother-bird are her nestlings....

*Semi-Chorus of Fathers:*

She is the Eagle of God!
Whose coming is sign of His blessing,—
Of the gift of Food to His children,
Of the gift of Life to His children,
As the mother-bird home circling
Beareth food and life to her nestlings....

*Full Chorus:*

Helpless are we as are nestlings,
Naked as unfledged eaglets
Lone in their storm-beaten crag....

*Semi-Chorus of Fathers*:

About them circleth the Eagle,
Strong to protect, ever watchful,
His plumes flashing white in the sunlight,—
The cloud-frothing winds are his coursers!

*Semi-Chorus of Children*:

Over them hovereth the Eagle,
She of the brown brooding pinions,
Bearing them food in her talons,—
As the Father of Heaven permitteth.

*Full Chorus*:

We men are as naked and helpless
As the storm-beaten chicks of the Eagle....

He of the wide-encircling Heavens guardeth us,
And the Sun-Father watcheth over us;
Mother Earth bareth Her bosom unto us,
Her bounty is our daily bread....

*Amid silence the Leader and the Acolytes retire through the Center Gate, as into an Holy of Holies. The Chorus remains ranged in the North and South Forefronts.*

THEME IV.

*The oboes and bassoons strike up the droning music of the Corn. The two Chiefs step forward.*

*Chief of the Fathers*:

Father, have pity upon our weakness,
Father, have pity upon our hunger:
We men are as infants before thee,
We men are as helpless children
Weeping for food....

*Chief of the Children*:

Out of far distant days soft-stepping,
I beheld one coming, a Spirit coming,
Coming to comfort me....

In the tender and caressing night
I beheld my comforter:

Her wings dropped the dews of fragrance;
With the softness of stars was her body beautiful;
In her breast were the singing voices of the fields....

*There enter from the North Gate two Acolytes, one bearing a tray of bread, one bearing a jug of water, and from the South Gate two, with bread and water. The Acolytes with the bread offer the bread to the Fathers and to the Children.*

*Chief of the Fathers*:

Lo, they bring ye the Body of Mother Earth:
Take thereof, and eat.

*While they partake of the bread, one after the other, the Acolytes with the water advance, offering the water to the Fathers and the Children.*

*Chief of the Children*:

Lo, they bring ye the Waters of Life:
Drink, and be refreshed.

*The Fathers and the Children moisten their lips as the water is offered them. The Acolytes pass forth as they entered.*

*The music becomes tense, vibrant and rapid. The Chorus sways to and fro in a crescendic rhythm.*

*From the Center Gate enter: Two Acolytes each bearing a staff with a spread eagle-wing attached; the Leader and an Acolyte bearing the plumed wands; an Acolyte bearing aloft the sheaf of maize; an Acolyte with the bowl. They range themselves, the Leader a little in advance and to the South, the Acolytes abreast, the wings at the ends, the Corn in the center, the white-plumed wand at the left hand of the Corn-bearer, the bowl at his right hand.*

*The Chorus, in animated motion, bursts forth in a lyric Hymn to the Corn*:

Daughter of Heaven, Earth's first-born,
Hail to thee! Hail to thee! Spirit of Corn!
Thou at whose bounteous feasts we are fed,
Who givest us life in giving us bread:
Hail to thee! Hail to thee! Spirit of Corn!

Thou who dost welcome the Sun-Father's glance
With tassel and spear flung aloft to His Morn,
With nodding of plume and waving of lance,
Thou who dost make the green gardens to dance
With joy of thee, joy of thee, Spirit of Corn!

Thou who dost gather the sunlight and rain
Till the body of Earth with Heaven is o'erlain,—
Life, life is thy largess, who givest us grain!
Daughter of Heaven, Earth's first-born,
Hail to the thee! Hail to thee! Spirit of Corn!

*They cease with nodding plumes.*

THEME V.

*The grave music of the Way of Life enters as an undertone to the Corn music. The Leader advances to the Altar. He signals to the Acolytes, who uplift the emblems. He addresses the Powers:*

Behold us, where we are standing,
Uplifting these emblems,—
Ye Mighty Ones, behold us!....

Out of the Heavens, cometh a flash!
Out of the Heavens, the light of His seeing eye!

*At a sign the five Acolytes, abreast, advance to the Altar, before the Leader. They present the emblems to the East, crying:*

Ye of the East, behold us!
Ye of the Dawn and the Day!

*They advance sixteen paces, wheel, and elevate the emblems to the West, crying:*

Ye of the West, behold us!
Ye of the Storm and the Night!

*They return sixteen paces, wheel to the left, advance eight paces south, and elevate the emblems to the South:*

Ye of the South, behold us!
Ye of the Path of the Sun!

*They wheel and advance sixteen paces to the North, elevating the emblems to the North:*

Ye of the North, behold us!
Ye of the Mother of Day!

*They return eight paces to their original station before the Altar, and once more advance sixteen paces to the East. There they remain, abreast.*

*The Leader advances eight paces from the Altar, till he stands, as it were, upon the heart of the human figure traced by the evolutions of the Acolytes.*

*The Leader*:

Ye of Heaven and Earth, behold us!
Ye Powers of Life, behold us!....
Who journey the way of man.

Ye have given us for our Strengthener, the Spirit of Corn:
Ye have given us for our Leader, the Spirit of Corn!....
Who journey the toilsome way.

As the Spirit draweth nigh, we bow our heads:
As the Spirit toucheth us, we bow our heads....
Who journey the Way of Life.

*While the music grows in depth and gravity, the Chorus moves, forming in a phalanx behind the Leader.*

*Then the Chorus*:

Open our way, Spirit of Corn!
Open our way, Leader in Life!

*The Leader*:

Open is the Way!
We are led as were our fathers led
Down through the ages:
We follow as they did follow.

*The Leader signals; the Chorus moves forward; the Acolytes, abreast, with the emblems upraised, the Corn still at the center, lead the processional, which circles the sward and finally retires, the Leader and Acolytes through the Center Gate, the Fathers through the North, the Children through the South. During this movement, in full choral, is sung the Chant of the Way of Life.*

I.

*During the advance*:

Follow on, O Brothers, follow on!
The Spirit of the Corn doth lead
And unto you at your need
Falleth her benison:
Follow on, O Brothers, follow on.
Whither your sires are gone....

Your feet one rhythm beating,
Your tongues one song repeating,
Your hearts one boon entreating,
Follow ye on!
Forth of the ruddy Morn,
Into the glowing Day,
Where the Spirit of the Corn
Showeth the way:
Follow on, O Brothers, follow on,
Whither your sires are gone.

II.

*Circling East*:

Lo. the Circle of the Earth
Is the circle of Man's domain,
And he buildeth his puny hearth
In the midst of her spreading plain,—
And Morning and Noon and Night
He kindleth his tiny light.

*Circling North*:

Heaven hath a myriad stars,
Heaven hath the burning Sun,
The Day and the Night are their bars,
And their course is never run:
In the hour where it began
Dieth light in the lodge of man.

III.

*Circling West*:

Man walketh in ways unknown,
From the darkening East to the West—
As a fledgling that hath flown
Forth from the Eagle's nest
To journey the pathless skies
With the sun of Heaven in his eyes.

*Circling South*:

Man bareth his head to the rain,
His breast to the storm layeth bare,
And he stalketh athwart the plain
Blind in the lightning's glare;

And heavy on his soul
Falls the terrible thunder's roll.

IV.

*Circling East*:

As an infant that is led
Amid the paths of surprise
By the hand that giveth him bread—
The hand of the foolish or wise—
So is a man in Their care
Who measure the ways he must fare.

*Circling North*:

The herds of the prairies pass,
At the will of the South and the North,
On the trail of the greening grass,
Where the Spirit of Life floweth forth,—
So man taketh up from the sod
The sacrament of God.

V.

*Withdrawing*:

Follow on, O Brothers, follow on!
In the ways whereto ye were born,
While leadeth the Spirit of Corn
Granting her benison:
Follow on, O Brothers, follow on,
Whither your sires are gone!....
Your feet one spirit guiding,
Your lives one fate abiding,
In the wisdom of One confiding,
Follow ye on!
Into the sombre Night,
Forth of the flashing Day,
To lands beyond your sight
Lieth the Way....
Follow on, O Brothers, follow on,
Whither your sires are gone.

[*Exeunt omnes.*]

## PART II. THE REVELATION.

### THEME VI.

*The music opens with an eerie prelude, full of whispering notes suggestive of things supernatural. The Chorus, as yet unseen, strike in with their Invocation to the Visions. They enter singing, the Children from the South, the Fathers from the North. They wear no bonnets and they are girt with blankets, symbolic of Night. They circle in opposite directions, passing and repassing, file by file.*

*The Chorus*:

Holy Visions, hither come!
Ye who dwell in rainbow Skies
Hidden from our mortal eyes
By the lights of Paradise,—
Holy Visions, hither come!

Holy Visions, hither come!
To our troubled lives descend,
Draw anigh and o'er us bend,
That our hurts may have an end:
Hither, hither come!

Holy Visions, hither come!
If we wake or if we dream,
Where your flashing pinions gleam
There doth Heaven on us beam:
Holy Visions, come!

Holy Visions, hither come!
Gift of joy your presence brings,
When the music of your wings
To the gladdened spirit sings:
Hither, hither come!

Holy Visions, hither come!
Glorified the spirit blooms
Where the splendor of your plumes
Like a sun its night consumes:
Holy Visions, come!

Holy Visions, hither come!
With the lightnings of your glance
Make the hearts of men to dance
In celestial radiance:
Hither, hither come!

Holy Visions, hither come!
Bearing with you Heaven's Peace,
Bearing every hurt's release
In your healing mysteries:
Holy Visions, hither come!

*The Chorus, as the song closes, form a semi-circle facing eastward, the Fathers to the North, Children to the South. They kneel and draw their robes over their heads, as in vigil. The two Chiefs stand, a little in advance.*

*The music is weird and mysterious, with innumerable fluttering crescendos, as of approaching wings.*

*Then the Chief of the Fathers*:

Hark, the sound of their wings!
Like the wings of mighty eagles:
Like the whistling winds on the prairies:
Like the rushing rain on the corn!....
Hark, the sound of their wings!

*A pause. Then the Chief of the Children*:

Was it in dreams that our Fathers beheld them?
In wingéd dreams that they came revealing
Unto our sires the Vision of Life?

*Chief of the Fathers*:

Yea, in their dreams our Fathers beheld them:
In shining dreams they came unveiling
Unto our Sires the Vision of Life....

*A pause. Then the Chief of the Children*:

Hark, the sound of their wings!
Mighty spirits hither flying:
Mighty spirits here revealing
Visions as in days of yore....
Hark, the sound of their wings!

*The mysterious music continues for a time; then dies away into the steady beating of deep-toned drums. An utter silence.*

THEME VII.

*A burst of drum-beats. The Chorus throw aside their robes from their heads, and rise, crying:*

Awake, O Mother, from sleep! The night is far spent.
Awake, O Earth, from your rest! The hills and the valleys stir.
Awake, O World, from your night! Day summoneth Earth and Sky.

*The Chorus moves, in a flowing rhythm, while the Chief of the Children sings the Song of the Dawning, the orchestra sustaining with a liquid and lyric mood:*

A Wind bloweth forth from the East,
The Wind of the wakening Dawn:
The clutches of Sleep are released
Where the Wind bloweth on, bloweth on....
The liquid Wind of the East,
The living Breath of the Dawn!

Lo, from her crag-built nest
The Eagle glanceth afar!
She preeneth her golden breast,
And with sweep of her pinions doth soar
Over the world's dim crest
Where the lights of the Morning are.

See! In the Eastern sky,
As a herald that runneth swift,
As a chieftain who draweth nigh
With ruddy plume uplift,
One cometh .... and passeth by:
The tidings of Dawn are his gift!

'Tis the Star of the Morn, of the Morn!
A runner whom none shall withstay,
Whose red-shining token doth warn,
As he courseth his luminous way,
That a Child from the Night hath been born:
The Dawn! who foretelleth the Day!

*To the growing animation of the music, the Chief of the Fathers:*

Behold!
A light in the East!
Behold!
The whitening Dawn!
Unto their morning feast
The creatures of light move on:
In pasture and brake
The world is awake
With browsing herd and with wilding deer:
The Day is here!

*The music is rapid and exultant. The Chorus is in swift, swinging motion, with imitative action suiting the words of their choral. All about is an incessant tinkling, as of castanets and little bells.*

*The Chorus:*

Day is here!
Day is here, is here, is here!
Day is here, is here!

Awake, awake! On the hills the light is breaking!
Awake, awake! The heavens are aglow!
The sleepers all, their coverts are foresaking;
The winds of morning freshen as they blow:
Athwart the plain the deer with antlers shaking,
Athwart the sky the singing wildwings go!
Awake, awake! While dewy Earth is making
The springs of life with morning gladness flow!

Day is here!
Day is here, is here, is here!
Day is here, is here!

*The song ceases with the Fathers to the North of the Altar, the Children to the South, all facing toward it, their parallel files forming a broad avenue from the Center Gate to the Forefront.*

## THEME VIII.

*The music becomes strong and broad, developing the motive of the creation of light and life and the mystery of revelation.*

*From the Center there enter, in single file: An Acolyte bearing the spread wings, carried as a banner; an Acolyte bearing the white-plumed wand;*

*an Acolyte bearing a tray upon which is fire and tobacco; at a distance, the Leader, as one inspired. The three Acolytes advance to the Forefront; the Leader remains before the Altar.*

*The two Chiefs with their calumets go before the Acolytes. They take the fire and the tobacco and offer a smoke-offering to Heaven. Then the Acolytes, in single file as before, withdraw through the Center; the Chiefs retire to their stations.*

*The Leader, with uplifted gaze, intones the Psalm of Revelation:*

I

*With brooding mystery:*

As I lay sleeping,
As I lay dreaming,
Out of the distance came one advancing,
Out of the distance came one descending,
As cometh a star from the deep of Heaven,
As cometh a star in a pool of light,
Welling to fullness,
Welling in stillness,
Till resteth its ray
On the brim of the World.

As I lay sleeping,
As I lay dreaming,
Out of the distance one came flying,
Out of the distance, with whirring of wings....
As I lay sleeping,
As I lay dreaming,
Over me drooped her glittering wings,
Over me drooped, while she chanted the mystic
Spell of the riddle that ruleth the World.

As I lay sleeping,
As I lay dreaming,
She sang me the Song of the Eldest of Mornings,
She sang me the deeds of the Father creative,
She sang me the cure of the leaderless life....
As I lay sleeping,
As I lay dreaming,
She read me the riddle that ruleth the World.

II.

*With austere solemnity*:

How they that were above were in Darkness
And they that were below were in Darkness:
When over all things brooded the Night, heavily....
Silent were all things,
All lay hushed.

Then the Father of Heaven breathed the Breath of Life;
Then the Father of Heaven moved upon the face of Darkness,
Upon the Body of Night,
Upon the body of the Mother of Day,
Moved the Father of Heaven,
Breathing the Breath of Life.

A Child to the Night is born!
Unto the Father of Heaven and unto the Night
Is born the Dawn....
Whose breath is the Breath of Life,
Whose gift is the Gift of Life
Unto all things.

A Child to the Night is born!
Yea, the Dawn,
Whose father is the Father of Heaven
And whose mother is the Night....
And all things above
And all things below
Are quickened into being.

III.

*As at first*:

As I lay sleeping,
As I lay dreaming,
She sang me the Song of the Eldest of Mornings,
She sang me the deeds of the Father of All.

IV.

*Solemnly, but with enthusiasm*:

Then the Father of Heaven created the Chieftain Sun:
Who is sire of the shining Day;

Who is leader of the Wardens of Light;
Who holdeth the measures of the years.
His spouse is our Mother Earth,
His warmth is the warmth of all that live,
Gladness is his offspring:
Whom the Father created Chieftain of the Skies.

Yea, the Father of Heaven united Earth and Sun
In Holy Marriage,
Whereof are born her breathing Children—
Bird and beast and mortal men—
And all her living fruits:
The Father of Heaven united Earth and Sun,
Whose Child is mortal Life.

V.

*As at first*:

As I lay sleeping,
As I lay dreaming,
Lo, in a Vision one came revealing
The Mystery of Life.

VI.

*With exaltation*:

Give heed! Give heed!
Give heed, O ye People!
Unto the Abode of Life give ye heed,
And unto the Powers thereof
Let your hearts be turned in reverence....

*The Leader remains beside the Altar.*

THEME IX.

*The Chorus moves in stately alternation of the Semi-Choruses, chanting their antiphon to Earth and Sun.*

*Semi-Chorus of Fathers*:

Now behold! Hither cometh the ray of our Father Sun,
Over all the land, us to touch and give us strength!

*Semi-Chorus of Children*:

We think on Mother Earth who lieth here:
We know she giveth of her fruitfulness.

*Semi-Chorus of Fathers*:

Now behold! Where mounteth up our Father Sun!
Into the Lodge of Heaven he mounteth up.

*Semi-Chorus of Children*:

Behold on Mother Earth the growing fields:
Behold the promise of her fruitfulness!

*Semi-Chorus of Fathers*:

Now behold! Through all the World our Father Sun
Sendeth his rays, the Messengers of Light!

*Semi-Chorus of Children*:

We think on Mother Earth who lieth here:
We see the promise of her fruitfulness.

*Semi-Chorus of Fathers*:

Now behold! How all the life of hill and plain
Is quickened by the rays of our Father Sun!

*Semi-Chorus of Children*:

Give thanks to Mother Earth for trees and streams;
Give thanks to Mother Earth for growing fields;
Give thanks to Mother Earth for ripened corn;
Give thanks to Mother Earth for food and life!

*Semi-Chorus of Fathers*:

Now behold! Where goeth down our Father Sun,
Who of his strength this day of life hath given!

*Semi-Chorus of Children*:

We think on Mother Earth who lieth here:
Truly, her power she hath given us!

*Semi-Chorus of Fathers*:

Now behold! Where sinketh low our Father Sun
Upon the margent of the glowing West!....
So is the life of man led forth
Out of the Night, through Morn and Noon and Eve,
To sink into the silent Night again!

*Semi-Chorus of Children*:

We think on Mother Earth who lieth here....

### THEME X.

*The mysterious music of the inaugural is resumed, but with a deeper, more austere meaning. The Chorus forms for the outgoing. Then the Leader, with arms outspread:*

There is none persuadeth Death!
The old men have not told how any hath found a way.
The career of a Leader is difficult!

*Marching counter, as in their entrance, the Fathers and the Children circle the sward and pass out at their respective gates, chanting:*

Holy Visions, ye of yore
To our Fathers came revealing:
Hither come, O come once more,
To our troubled lives with healing!

Holy Visions, ye who bring
From the starlit Sky her glories,
Hither come on shining wing,
Pause ye where the open door is:

Pause ye at the open gate,
Enter at the silent portal,
Bless the hearts of them that wait
With the grace of light immortal:

With the grace of holy sight
To the dream-life of the dreamer
Ye shall come, and guide aright:
He shall know his life's redeemer.

Holy visions! As of yore
To our Sires ye came revealing,
Come, O come to us once more,
With the mystery of healing!

[*As the last of the Chorus is disappearing, the Leader retires, solitary.*]

## PART III. THE MYSTERY.

### THEME XI.

*From the North and South Gates the Fathers and the Children, except their Chiefs, who remain behind, enter in an animated and swift-scattering*

*movement, giving the effect of individual wheelings and circlings and poisings over the whole plaza. The music is lively and full of bird themes.*

*The Chorus*:

Hark, hark! The birds!
The birds are a-wing!
Earth and Sky are alive
Where they flit, where they swing!
Where they dip, where they dive,
And down the winds drive,
Till with whir and with whing
Of thunderous wing
The volleying air
Is a-blare, is a-blare!

Rising, circling, dipping, fleeting,
Now they rest, and now they haste!
Coming, going, parting, meeting,
Bird to bird his cry repeating:
"Summer nest is Wintry waste!
"Winter stealeth Summer pleasure,
"Garb of green he turneth gray:
"Where the winds bear Summer's treasure,
"Thither, thither, haste away!"

Flutt'ring, flocking, flitting, flying,
Now they rest, and now they haste,
Bird to answering bird a-crying:
"Summer nest is Wintry waste!"

*Individual singers, one by one, sing the songs of the birds, with mimetic action.*

*The Song of the Nestling*:

O'er the prairie, o'er the prairie,
Round about me as I walk,
How the shadows flit in circles—
Mischief shadows, making mock!

'Tis the birds above wide circling,
'Tis their shadows on the ground:
As when parent birds protecting
Feeble nestlings circle round.

Birds of Heaven, Birds of Heaven,
We, your nestlings, joyous cry
When His sign of care ye give us,
Wheeling in the azure sky!

*The Song of the Wren*:

Whe kee re re wee chee!
Whe kee re re wee chee!
Joy, joy, joy!
Singeth the tiny Wren:
And shall not men
Know joy?

*The Song of the Duck*:

Lo, the Finder-Out of Ways—
The Bird of the Emerald Crest—
The Bird who never strays,
But doth fare
In arrowy flight and ware
Over water and earth and air,
North and South,
East and West....
Oh, the speeding Scout of the Skies
Knoweth their quartering ties:
As the Leader of Men must know
Where the paths of Heaven go!

*The Song of the Owl*:

He! Hiri Wahoru!
He! Hiri Wahoru!
Wide-eyed Bird of the Night,
Who seest invisible things
And spreadest thy shadowy wings
In dim and inaudible flight....
He! Hiri Wahoru!
He! Hiri Wahoru!
Let ours be the gift of thy sight!

*Full Chorus*:

Oh, the Bird, the Birds!
The Birds are a-wing!

Like sky-blown herds
At the wintry sting
Which the North
Striketh forth....
Where they come,
Where they go,
All the air
Is a-blare,
All the air is a-thrum,
As with beating of drum
And sounding of string
Where drawn is the bow
And the swift arrows sing!....
Oh, the Birds are a-wing!

Summer flown,
Nestlings grown,
Southward blown
Wide a-wing!

*The Chorus ends its evolutions with the two divisions forming, as it were, encircling wings, across the Forefront, and facing Northwest and Southwest, so as to view the gates.*

THEME XII.

*The temper of the music becomes more grave, with the flutes of the Eagle dominant.*

*Semi-Chorus of Fathers*:

Behold, an Eagle now is circling, widely circling above us!

*Semi-Chorus of Children*:

As the mother-bird circleth her nestlings, careful for her chicks,
She circleth us, hovering....

*Full Chorus*:

She is the Eagle of God!
Of Him who is Father of Heaven,
Who ruleth the zonéd Earth
And sendeth His will by the Eagle
Over the windy Pathways
That lead from Man up to God!....

*The motive of the music is the poignancy of human aspiration.*

*From the North Gate, the Gate of the Fathers, enter: An Acolyte with the Spread Wings, borne as a banner; an Acolyte with the Corn, one with the Bowl, one with Tray and Cups; the Leader, with wand; an Acolyte with the white-plumed wand. They march in single file, their path a semi-circle from the North to the South Gate, around the Altar. At the Altar they stop.*

*The Leader*:

I know not if the voice of man can reach unto the Skies;
I know not if the Silent One can hear me as I pray;
I know not if my words be foolish words or wise;
I know not if I walk in straight or crooked way.

I only know His power, Who hath made our mortal lot
An hurt and stumbling pace led outward through the dark;
I only know his trust, Who lest He be forgot,
Hath weathered deep the soul of man with an immortal mark.

*As they move on toward the South Gate, the Gate of the Children, the Acolytes sing, in choral*:

Father, unto thee we cry!
Father of all we hear and see,
Father of all we feel and hope,
Author of life's mystery:
Father, unto thee we cry!

*They pass out.*

THEME XIII.

*The Fathers, pianissimo bass, sing*:

With the dawn will I seek my child,
With the tenderly growing dawn;
Where the breath of the morn floweth on
I will go seeking my child,
My little one, my son....

*With swelling music, the Children*:

Father, come unto me here,
Here where I wait for thee,—
With bread and with morning cheer,
Father, come unto me!

*The Fathers*:

I come, my child, I come,
Seeking for thee....
Abide me, and nothing fear:
On the wings of the dawn I come
Seeking for thee....

*The Children*:

See!
The Eagle is flying o'er us!
In the sky above, from the Father's home!
The Eagle descendeth unto us
With the Father's cheer!

*In the music is the note of the dawning Light. Then the Chorus*:

Behold!
The Star of the East!
The Star of the bursting Morn!

*From the Gate of the Children a runner, personator of the Morning Star, clad all in red, the color of life, and in his hair a red plume, symbolic of the breath of life. To his arms are attached spread wings. Sweeping past, he cries*:

A Child is born!
Unto Man a Child is born!
Unto Man is born a Son!

*He passes forth by the North Gate, the Gate of the Fathers.*
*The Chorus*:

A Child is born! A Child is born!
An holy Child is born!
Stars of the Morning rejoice!
Life is renewed in the World!

*The music swells with prophetic exaltation.*
*Enter from the Holy Gate: The Acolyte with the Spread Wings; the Acolyte with the Corn, he with the Bowl, he with the Tray on which are the four cups; the Acolyte with the white-plumed Wand; the Leader; the Chief of the Fathers, carrying the Child; the Chief of the Children.*
*When all are entered the Leader takes the Child and holds him aloft, crying*:

Behold the Child!

*The Chorus*:

Behold the Child!
Behold the Promised One!

*The Leader returns the Child to the Chief of the Fathers, on either side of whom the Acolytes range themselves, and leads the way to the Altar, while the Acolytes sing*:

Here we go singing, singing,....
Looking on the Child—
The little Child who leadeth us,
Borne in his father's arms:
Here we go singing, singing....
Looking on the Child.

THEME XIV.

*At the Altar they form: the Leader a few paces in advance, at his left the Chief of the Fathers with the Child and the Chief of the Children; the Acolytes ranged before the Altar, the white-plumed Wand to the North, the Spread Wings to the South.*

*The Leader spreads his hands, like spread wings, above the Child. He signals to the Acolyte with the tray and cups, who advances. The Leader dips his finger into one of the cups and touches the Child, drawing a semi-circle about his brow.*

*The Leader*:

With the Blue of the Skies I anoint thee....

*The Chorus*:

That thou may'st long abide beneath the Lodge of Heaven.

*The Leader dips his finger into a second cup and draws it across the Child's chin*:

With the Green of the Earth I anoint thee....

*The Chorus*:

That thy feet may be led amid fruitful fields.

*Dipping into a third cup and touching the Child's cheeks*:

With the Crimson of Life I anoint thee....

*The Chorus*:

That strength and vigor shall be thine in youth and age.

*Dipping into the fourth cup the Leader touches the Child's brow*:

With Oil and with Fat I anoint thee....

*The Chorus*:

That peace and plenty may follow thee all thy days.

*The Acolyte retires; the Leader once more spreads his hands above the Child; a second Acolyte advances, bearing the Corn. The Leader taking it, strokes the Child's body*:

I stroke thee with the ripened Corn....

*The Chorus*:

So may thy body's needs be satisfied!

*The Acolyte with the Bowl advances. The Leader sprinkles the Child*:

I refresh thee with the clear and running stream....

*The Chorus*:

So may thy generations run onward without ceasing.

*The Acolytes retire. The Leader takes from his hair the white eagle-down and fastens it in the Child's hair*:

With this sacred token I thee adorn—
Symbol of the fleecy clouds above,
Symbol of the winds of Heaven,
Symbol of the living breath
Into the body of man
Breathed by the Father....

*After a pause, his hands resting on the Child's head*:

Enter ye into the House of Life, consecrate.

*He returns the Child to the Chief of the Fathers. Then triumphantly*:

I know now that the voice of man can reach the skies;
I know now that the Mighty One can hear me as I pray;
I know our Father answereth his children's troubled cries,
And pace by pace assigneth us the token of the way.

Give heed! Give heed!
Give heed, O ye People!
Unto the Abode of Life give ye heed,
And unto the Powers thereof
Let your hearts be turned in reverence....

THEME XV.

*The music becomes reminiscent of the Chant of the Way of Life. The Chorus moves forward, forming a circle, the Children within, the Fathers without, as in the figure of the lodge. The Leader advances beyond the Altar and paces a small circle, or symbolic lodge. The two Chiefs enter this circle while the Acolytes, with emblems raised as in blessing, form a semi-circle behind.*

*The Chief of the Children takes the Child from the Chief of the Fathers. Then the Chief of the Fathers moves forward and sings:*

Within the House of Life man entereth
A little Child with slow and faltering feet:
The breathing Heaven is in his fluttering breath,
The pulse of Earth in his swift blood doth beat.

Within the House of Life man tarrieth,
As one who for a season taketh rest:
The Blue above, below the grassy Earth,—
An oriole within a wind-swept nest.

Within the House of Life man offereth
The simple tokens of his daily need,
His prayer for food and drink, in humble faith
That some dim distant Power shall give them heed.

Then from the House of Life he hasteneth....
Aye, as an Eagle in his feathered mail
Battleth adown the blast with windy Death,
Speedeth the Warrior-Soul with battle-hail!

*The Chorus is in motion, moving in a strange dance simulating the flight of eagles. They form in files and circle about the central group. They sing:*

Come, ye Fathers!
Come, ye Children!
Come, ye People,—
Mortal men!
Into the House of Life, come enter!
Into the House—the Way is open:
Enter in, O mortal men!....
Like flocking birds,
Like shouting eagles,
Full of joy and lust of life,

Swiftly, swiftly, swiftly come ye,—
Enter in, O mortal men!....
As your Fathers came before you,
As a little child doth come,
Where the Way is open, open,
Enter in, O mortal men!....

*As they cease, the two groups, the Fathers and the Children, are formed, on the North and the South, like the spread wings of an Eagle. The two Chiefs, in the center, are the bird's body; the Acolytes, with the emblems, have retreated, forming, as it were, the tail plumes; the Leader, with the Child, has advanced to the head.*

*There falls an utter stillness. The Leader uplifts the Child, looking upward. In a penetrating voice he cries:*

Breathe on him!
Breathe on him!
Life thou alone canst give him:
Long life, we pray, O Father, give unto him!

*Mid swelling music, like the march of the tribes and nations of men, exeunt omnes.*

THE END.

# CRITICISMS AND DISCUSSIONS.

## BERGSON AND RELIGION.

Henri Bergson is probably the most potential name in modern philosophy. Prof. William James, who was by common consent our most distinguished thinker, though he was much older, called Professor Bergson "master and teacher." This, certainly, is high praise.

Aside from his speculative capacity, Professor Bergson is a most interesting figure. He is an earnest student of physiology, biology and psychology, and he brings to his philosophical theories a great wealth of scientific illustration and proof. And unlike so many of our great metaphysicians, he has literary power, the gift of musical speech. Whether the *Evolution créatrice* is great art like the Corinthians of Paul, the Divine Comedy, "Lycidas," or "Les Miserables," it may be too soon to decide. But it is certainly a work of art, and of no mean order. Professor Bergson is a personality, and his thought is always suggestive and commands attention.

It is interesting to watch the flight of his speculative arrows, even though we fail to see that they strike any target. Nevertheless, in my judgment he has made one vital suggestion, which I shall indicate in the course of this study. But first I shall attempt to trace his theory of the universe and his theory of truth and show their philosophical and religious meaning and influence.

As every one comes to a study with certain prepossessions, I may say that I am not a materialist, idealist or pragmatist, but conceive there are in man elements not mechanical and that he has, within narrow limits, the power of choice.

Professor Bergson in his theory makes an immeasurable primeval "super-consciousness" the source of all things, of life and matter. This unique creative absolute has will, freedom, and an impulse to create, but strange to say, though it has this consciousness and spontaneity, it has not intelligence. It moves on and on, ever unfolding, ever augmenting, with no design or purpose, seeking

no predetermined goal, for M. Bergson frowns upon all forms of teleology.

This theory of a great life-river, if I may so describe it, ever seeking to find new channels of creative opportunity, I found to my surprise was similar to that of my friend, Prof. F. C. Doan, published in the *Journal of Philosophy* about two years before the "Creative Evolution" appeared. I learned, since commencing this paper, that Professor James had made the same discovery. And I may say that, leaving off certain naive features in the book of Genesis, M. Bergson's theory of the origin of the world reminds me of that great sentence: "In the beginning, God."

Whence comes this vast energy with its impulse to create, M. Bergson does not tell. He asserts that from it spring both life and matter and that every living thing, from the lichen on the rock to the golden dandelion nodding in the south wind, from the ameba to the man, possesses consciousness and freedom, and these qualities enmeshed and entangled in matter, reduced or attenuated to the finest threads, are never lost. At times Professor Bergson calls matter "the enemy" of all good. It is ever to be resisted, it must be transmuted into living organisms, it must be saturated with "contingency."

Again he calls the resistance of matter a "stimulus." It is by the reduction of the flesh, by the chastening of the senses, that men become healthy, strong and beautiful.

It will be seen that in this dogma of the life-urge, M. Bergson strenuously opposes the new naturalism so popular at the close of the last century. He affirms that life always has in it the seeds of freedom or contingency, that contingency grows greater as organisms develop. He cannot believe that the high reason that has traced the laws of the earth and measured the stars, that the hope, affection, imagination which blossomed into the melodious words of the Sermon on the Mount are the product of mechanical and unintelligent forces—that blind physical atoms could in time stumble into an orderly living universe!

Has Professor Bergson spoken a deep, living word? Has he made a new synthesis? There are many who believe that he has. He opposes the older idealism of Kant, Hegel and Fichte, and the "absolute" of such teachers as Royce and Bradley in his theory of time. For unlike them, he makes time a reality, and in time creation begins. His theory of this original creative energy makes the universe of life and matter a great Mississippi life-river, ever flowing

on. Its course may be traced in the past and in the present, but its course in the future, he says, no one, however wise, can trace.

That the future is impenetrably veiled (an idea I have long contended for), Professor Bergson urges from the fact that the universe is not made but making. There is ever the condition of uncertainty, of spontaneity, of contingency, and thence may come the unexpected. We see now the leaf, the stalk, the bud, but of the glory and beauty of the flower and fruit, none can know.

M. Bergson's opposition to materialism is seen in his radical idea of freedom. He maintains that in all living organisms there is something that cannot be accounted for by the laws of matter. There is in them a power which draws from itself more than it receives, "gives more than has been given to it." There is something free in the violet, the bird, the man, not produced by reflex action. There is a tiny will, a drop of beauty, of will, of love, of intelligence, which is pure creation. There is ever the quality of the contingent, the new, the unforeseen, for this is a "spiritual universe."

Of course the idealist will say that Professor Bergson's theory destroys the timelessness and omniscience of the Absolute, and the naturalist will ask for proof. He will inquire, why it was necessary to invent a "superconsciousness" to start the universe. He will say it is just as easy to think of life evolving from matter, as matter from life; and the idealist will be alarmed at the thought of admitting into the universe the element of imperfection and the unforeseen.

But we must now come to the more original, and more radical part of his theory, his theory of truth. The extreme radicalism of his idea may be seen from the fact that M. Bergson makes "not reason but instinct bring us into the closest touch, the directest relation with what is most real in the universe," to use the words of Mr. Balfour. In this, I may say that Professor Bergson follows the present tendency to distrust the power of the intellect to reach a reasonable explanation of the universe—to prove the existence of God, of freedom, of immortality. All questions of ultimates are beyond intellectual search. The intellect is limited to the sphere of experience.

Professor Bergson agrees with this distrust of the intellect, but affirms that what is impossible to the intellect is possible to instinct. The province of reason is not life, freedom, spirituality, but matter, mechanics and space, "the waste products of the" life-urge. James agrees with his teacher here, for he says that "the reason can know

only surfaces." But, one may ask, are not these statements purely dogmatic, speculative?

Professor Bergson, while he admits the immeasurably wider horizons of the human intellect, asserts that instinct, in ants and bees in which it comes to its perfect flower, is in touch with a higher order of truth. Maeterlinck makes a similar assertion in his work on the bees.

But surely there lie innumerable difficulties in the path of this fantastic theory. If the instinct of the Hymenopterae is the infallible organ for the discovery of knowledge, why is it that they do not advance, but keep in the same monotonous round? With this great power, why is their achievement so limited, their vision so narrow? Why should they have so much of this divine power, and man, who is so incomparably greater, have so little? With this great endowment, wherein have they advanced beyond him?

Professor Bergson tells of a certain kind of wasp, the fossorial, which, instead of killing its victim, stings it into unconsciousness by a most delicate surgical act. This mechanical skill, he says, does not come as the result of numberless experiments, and it would be forever impossible to intelligence, but it comes through that instinct which reveals to the wasp the secret of life itself.

Does it not seem fantastic, to say the least, that the instinct of the fossorial wasp can reach a higher truth than the most sustained efforts of a great intellect? Can the work or conquests of the ants and bees compare with the magnificent achievements of the human intellect in mathematical, physical and moral science? Do we come into nearer touch with reality in the cell of an ant than in a painting by Titian?

It is true that the instincts are nearest the primeval forces, and may guide us best in the things of the flesh. Instinct may, by a sort of divine unreason, go straight to the heart of the lower truth, but to solve the supreme problems, the meaning of life, the existence of God, of freedom and of immortality must be an achievement of the highest intellect.

But it would not be fair to M. Bergson, not to explain that these mystical assertions, these speculative dreams, are enmeshed in a profusion of scientific illustration. He shows a minute and wide knowledge of physiology, biology and natural history, and in his boldest speculative flights always makes his final appeal to concrete facts.

But now I come to the question that will arise in many minds:

Is the philosophy of Professor Bergson religious in the highest sense? Does it make its appeal to our spiritual faith and aspirations?

Though this philosophy is radically opposed to the mechanical and atheistical tendencies of naturalism, many will say it cannot be called religious, as M. Bergson certainly means it to be. It is true, the Christian may see theism in the primeval life-urge, which is the source of matter and all living things, and in the exaltation of the instinct a recognition of the validity of the religious intuitions; but it will be difficult for the intelligent man to see a real theism in this primeval creative consciousness, though it has the will to create and freedom, but has no plan or purpose, nor directs the universe to any intelligent goal.

And while in his theory of evolution he escapes the difficulty or dilemma of the old metaphysical systems (that the imperfections, the evil, the sorrows of the universe, had been known to God before He created it, and were of His own selection), it does seem difficult to feel the sense of worship in the thought of a universe ever evolving, yet ever unintelligible and unmoral.

In the pluralism of Professor James, though he calls himself a pupil of Bergson, there is something for the common mind to catch hold of. When he says that God is the deepest power in the universe and is a personality, that "man and God have purposes for which they care and each can hear the other's call," he makes an appeal to the humblest believer. But I fear that the common people will not see the religious element in the philosophy of Professor Bergson. The saints who love and pray will cling to the thought of a transcendent God, leading the world to a wise and happy end, rather than believe in this impersonal life-force that forever unfolds, goes on and on, but knows not whither it is going.

On the other hand, the scientist will have his own thoughts. He sees that M. Bergson, to find an explanation, goes back to that primal sea of life. He will say that he cannot discover wherein that is different from the theologian's going back to God.

Yet, on the whole, I should say that the philosophy of Professor Bergson is theistic rather than atheistic, and spiritual rather than material and mechanical.

I said in the beginning of this study that Professor Bergson had made, in my judgment, a vital suggestion, and that is his recognition of the high function of philosophy. Although in his theory he remands the intellect to a much lower place than instinct, he forgets it in practice when he affirms that the vital, the supreme ques-

tions, "What are we; What are we doing here; and whence do we come and whither do we go?" *are the very cause of philosophy's existence*; and that the *future* (italics mine) will give back to philosophy its rightful place—the first.

Professor Bergson does not think that we can arrive at objective certitude or that we can force assent, but he suggests that the collection of many facts and their interpretation may give us a direction, "a direction only." These "lines of facts" will give nothing but a probability; "but all together, by converging on the same point, may give us an accumulation of probabilities which will gradually approximate *scientific certainty*."

It is a pleasure to see the view I have been contending for—that to this present discredit of the intellect, of philosophy, there will come a reaction—confirmed by the high authority of Professor Bergson. How far the reality to be known may exceed the power to know I cannot tell, but this seems reasonable, that the universe has an intellectual answer to those intellectual questions with which it continually confronts us. There is in us the indomitable belief that the terror and mystery of the material world may be transformed by a large knowledge into "transparent formulae." Should we not have the same belief that the terror and mystery of the moral and religious worlds may be also, by a larger intelligence, transformed into "transparent formulae"?

My study must end here, and I am aware how imperfect it has been, but I have tried to represent Professor Bergson kindly and impartially. This task has not been easy for, as Mr. Balfour says, there are parts of his theory, especially his theory of knowledge, difficult to comprehend; but I am sure all will consent that he has broken open new ground, and we can admit even the exaggeration of Professor James: "Open Bergson and new horizons loom on every page you read. It is like the breath of the morning and the song of birds."

JAMES G. TOWNSEND.

JAMESTOWN, N. Y.

## THE ANTI-INTELLECTUAL MOVEMENT OF TO-DAY.

Never before in the history of the world has science played such a prominent part and received more recognition as the main factor of civilization. And in truth there is a general agreement as

to the hope that we stand at the threshold of the age of science, which means that all problems of life will be solved by scientific inquiry and the old superstitions will be swept away. This principle has been applied to the several domains of life, to transportation, to sanitation, to the preparation of food and medical problems, the building of our homes and public edifices, yea even to the sphere of social and religious life. It is strange, however, that in these very days there have repeatedly appeared philosophical movements which are decidedly anti-intellectual, and treat science with a contempt in favor of the instinctive promptings of sentiment, which is only paralleled among the most old-fashioned dogmatists, in the tendencies of religious faith by such men as Augustine and Luther who treat reason as an enemy to faith, and endorse the old principle *Credo quia absurdum.*

We will here make a few comments in explanation of this movement without taking sides either with the admirers or the critics of the new fashion. The latter, the critical aspect, is most exactly represented by Mr. Bertrand Russell, pages 321 to 347 of the present number of *The Monist*; the other to some extent by the Rev. Dr. James G. Townsend. Mr. Russell points out that "if he (Bergson) fails in his condemnation of the intellect, the intellect will succeed in its condemnation of him."

It seems rather strange that in the days of the dawn of an age of science such movements should be so prominent, but it seems to me that these movements are the natural reaction against the many wrong aspirations of science, for it can not be denied that the prominence which science has gained in our days has also produced a number of narrow-minded scientists, who apply their narrow view of science to the whole of life. To them science is either physics or chemistry or biochemistry, or whatever their specialty may be, and most of them are acquainted with science only in its lower branches, mechanics or physics or some other domain which is void of the higher development of man where it unfolds itself in social and moral ideas. Psychology to such minds is a mere function of the brain, and the truly typical features of the soul are an accidental by-play of its coarsest substratum, or to draw their ultimate conclusion, mind is considered a function of matter. Their view of nature is limited, and while they rob man of his nobility they degrade him into an equality not only with the brute but even with inanimate existence.

The expression of this kind of narrow-minded science which

is not true science but the lowest step in the development of science, has caused a distrust in the true nature of science.

Anti-intellectualism has become fashionable in the philosophical world. Prof. William James made a great propaganda for it and succeeded mainly by his amiable personality. He speaks in the name of a certain common sense which stands up for unscientific views and defends a pluralism as well as a subjectivism on the ground that it is based on experience. For the same reason theory is discredited for sheer love of single and unrelated facts. Facts, however, are replaced by interpretations of a very primitive kind, among which even belief in spirits plays an important part. This incoherent system which abhors all moralism and actually represents a reaction to the world-conception of savage life goes under the name of pragmatism. It has made many conquests and gained many brilliant adherents even in the stable circles of European scholarship.

Another center of anti-intellectualism has been established in France of which Henri Bergson has become the leader through his unprecedented brilliancy of style and oratorical talent. He has gained many adherents in his own country, France, and celebrated high triumphs in conservative England. He is expected in the United States, and we have no doubt that he will be welcome in the circles of all who are dissatisfied with the quiet and unpretentious method of patient inquiry and scientific research. Men of this type possess great zeal and they will naturally welcome an ingenious representative of their philosophic tendencies.

In the meantime the spirit of criticism is stirring in England, and we have before us a book which with all soberness reviews the significance of the new star which has risen on the philosophical horizon. It is written by Hugh S. R. Elliot, LL. T., the editor of *The Letters of John Stuart Mill.** Sir Ray Lankester, K. C. B., F. R. S., being invited to write a few words by way of preface to Mr. Elliot's book, says:

"I am glad to do this, not merely because I think that the books in which M. Bergson formulates those illusions are worthless and unprofitable matter, causing waste of time and confusion of thought to many of those who are induced to read them, but also because an unmerited importance has been attached to them by a section of the English public, misled by the ingenious and systematic advertise-

* *Modern Science and the Illusions of Professor Bergson.* By Hugh S. R. Elliot. London, Longmans Green & Co., 1912. Price $1.60 net.

ment of M. Bergson by those who amuse themselves with metaphysical curiosities. He has been introduced to us as a "great French philosopher.' To those who in a thoroughgoing way occupy themselves in collecting and comparing and classifying all the absurdities which have been put forward as 'metaphysics' or 'metaphysical speculation' since the days of Aristotle, this latest effusion has, no doubt, a kind of interest such as a collector may take in a curious species of beetle. To the student of the aberrations and monstrosities of the mind of man, M. Bergson's works will always be documents of value. But it is an injustice as well as an inaccuracy to speak of their author as 'great,' or 'French,' or a 'philosopher.'....

"A main objection to M. Bergson's account of his own performances in the dark chamber [of the metaphysical *x*] is that he is not content with asserting (and expecting us to accept his bare assertion) that time is a stuff both 'resistant and substantial,' that consciousness is not always dependent on cerebral structure, that intuition is a true guide and the intellect an erroneous guide. Such escapades in the dark room astonish and interest only those who are unacquainted with M. Bergson's numerous predecessors in the maddening hunt of the illusive black cat. It is, however, a speciality of M. Bergson that having by mere assertion attempted to make us believe that he has grasped the black cat, and at any rate has in his hand some hairs from its tail—he proceeds in the same spirit to make absolutely baseless assertions about the domain of scientific fact—a domain 'tabooed' against him and his fraternity. He writes of the facts of physical science with the same careless assurance as that which we tolerate with indifference when he is disporting himself in the extra-territorial region of *x*. Having made his arbitrary assumptions about *x*, he proceeds in an inaccurate way to write about some of the well-ascertained facts of the structure of animals and plants. He promulgates novel opinions about them with the air of one who has given serious study to them, which, however, it is abundantly evident he has not. By a light-hearted perversion of the facts as to the structure of the eyes of animals and other such things, he endeavors to make them appear as evidence in support of his arbitrary and preposterous fancies about *x*! In doing so he ceases to be merely an amusing juggler with the harmless creations of his own and other people's fancy: he becomes a maker of untruth, and for those who listen to him a harmful *Confusionsmeister.*

"M. Bergson is gifted with an admirable facility of diction, and has succeeded in arresting attention. On that account, since he

has exceeded the limits of fantastic speculation which it is customary to tolerate on the stage of metaphysics, and has carried his methods into the arena of sober science, it is a matter of urgency that his illusions and perversions should be exposed with uncompromising frankness to the reading public who may be, on their side, under an illusion as to the importance of his teaching. Mr. Elliot's book effects this exposure in a masterly way."

M. Bergson proposes the strange doctrine that perception does not reside in the brain of the perceiver, but in the object perceived, —a proposition which is bewildering, and among his arguments he declares it theoretically not inconceivable that matter should be perceived without sense organs. Such doctrines belong to the cornerstone of his philosophy, and as an instance of Mr. Elliot's critique we will here quote some paragraphs discussing M. Bergson's theory of pain and of memory. M. Bergson defines pain as an "effort to repair damage." Mr. Elliot writes:

"Just as perception is located in the perceived object, so Bergson alleges that pain is located in that part of the body where it appears to be felt. This is of course in opposition to the belief of physiologists, who affirm that the pain is really located in the brain, not at the nerve endings; and who support their contention by pointing, for instance, to the pain which a patient feels and refers to his foot after it has for years been amputated. I am not, however, concerned to defend a well-established fact: I wish only to point out Bergson's mode of refuting it. 'If [the pain] is not at the point where it appears to rise, neither can it be anywhere else: if it is not in the nerve, neither is it in the brain; for to explain its projection from the center to the periphery a certain force is necessary, which must be attributed to a consciousness that is to some extent active. Therefore, he must go further....' Here we get a chain of deductions, every ling of which appears to be false. Why should any force be necessary? Why should that force be attributed to a consciousness? Why should that consciousness be active? It was one of Huxley's chief gifts to biology to have largely banished deduction from that science, by strongly insisting on the danger of traveling outside ascertained facts. A succession of deductions like this, in a physiological inquiry, is *a priori* almost certain to be erroneous. To me *a posteriori* there seems not even *prima facie* evidence in favor of any of them: and they are set against a fact experimentally arrived at!

"The doctrine of two kinds of memory is a complication of

natural facts that will not appeal to anybody. But the fundamental objection to it is that so often raised already: that there are no facts to support it. The Professor attacks the physiological view of memory: he adduces a number of facts, such as those of sensory aphasia, in opposition to it; and having destroyed it to his own satisfaction, forthwith we are presented with a new theory which is assumed to be true. This new theory is worked out in extreme detail; it is unaffected by sensory aphasia, but otherwise the only credentials it can produce are those of extreme unintelligibility. We have already had occasion to observe that a doctrine is safest from criticism when it is most difficult to understand. The fog is so thick that the critic is disarmed. I therefore make no specific attack upon it, beyond insisting upon the complete absence of evidence. Moreover, the attack on the physiological theory could scarcely convince ony one but a metaphysician. 'If memories are really deposited in the cortical cells, we should find in sensory aphasia the irreparable loss of certain determined words, the integral conservation of others.' But it is not so. Now, what would a man of science consider himself entitled to deduce from this? Nothing further than that words are not represented in the brain in minute specific areas for each word, but that they are, or may be, represented in some other way, possibly still undiscovered. But what does Bergson infer? That the memories of words are not stored in the brain at all. He refutes a crude physiological hypothesis; he then assumes that the refutation applies to all possible physiological hypotheses, and thence jumps to his own theory. It would have been just as reasonable to found his own theory upon a refutation of Gall's phrenology. For phrenology was a thoroughly materialistic hypothesis; it assumed absolute connection between mind and brain, and definite localization of mental faculties in the brain. Phrenology has long been exploded, but no one (except a metaphysician) infers from that that there is no connection between mind and brain. A belief in that connection is in no wise shaken by the exposure of phrenology; nor is it shaken by the criticism of other crude attempts to localize mental qualities. These criticisms are effective only for the particular theories against which they are levelled. Hence we see that Bergson's theory of mind and matter is founded upon the same fallacy as that of the vital impetus—the fallacy which we stigmatized as the mannikin fallacy at the beginning of the chapter. In bald outline it is like refuting Mahommedanism, and then arguing: (1) Mahommedanism is untrue; (2) therefore all religion is untrue; (3) therefore all

morality is a superstition. We have only to point out that (2) does not follow from (1), nor does (3) follow from (2). In Bergson's works the second step (2) is invariably taken silently immediately (1) has been established. The great show of facts in his works are all connected with step (1), the criticism of adverse theories. Step (2) is then slurred over without a word of discussion, and the rest of the philosophy is taken up with step (3), which is just a hypothesis or guess, or intuition, having no connection with foregoing facts, but set out with such a wealth of words and analogies that the unwary reader quickly loses his way and is totally lost. In alliance with the main paralogism is the copious misuse of analogies and of words, the latter especially in the form of materializing abstractions such as time, life, motion, memory. The medieval realists could scarcely have gone farther.

"The tendency to attribute substantial reality to abstractions is conspicuous not only in metaphysics but in the thinking of all primitive races. Thus a Basuto will not walk by a river lest his shadow falling on the water should be seized and devoured by a crocodile. Nearly all children at one time or another attempt to evade their shadows by jumping or running. Names likewise are looked upon as material things: as among the Chinooks, one of whom thought that Kane's desire to know his name proceeded from a wish to steal it. Here, as elsewhere, Bergson does nothing more than systematize and magnify, on an enormous scale, almost universal vices of thought."

As an example of Bergson's method we will quote a few more passages from Mr. Elliot's book. Bergson says:

"'Instinct is knowledge at a distance. It has the same relation to intelligence that vision has to touch.' Why, then, do we owe our knowledge of the stars to intelligence, and not to instinct? Why has astronomy advanced by the gradual triumph of intelligence over bigoted superstition?....

"Bergson's attempt to establish the preeminence of men and hymenoptera takes, in one place, the following form:—'It is unquestionable that success is the most general criterion of superiority, the two terms being, up to a certain point, synonymous. By success must be understood, so far as the living being is concerned, an aptitude to develop in the most diverse environments through the greatest possible variety of obstacles so as to cover the widest possible extent of ground. A species which claims the entire earth for its domain is truly a dominating and, consequently, superior

species. Such is the human species, which represents the culminating point of the evolution of the vertebrates. But such also are, in the series of the articulate, the insects, and, in particular, certain Hymenoptera. It has been said of the ants that, as man is lord of the soil, they are lords of the subsoil.'

"Under this definition, birds ought to be a dominating group, for their distribution is wider than that of men. And the most pre-eminent species of all would not be men, or insects, or even birds, but those simple unicellular creatures like ameba, which are found everywhere all over the earth."

Mr. Elliot sums up the whole book with the following conclusion:

"Professor Bergson's philosophy is contained in three volumes. I here summarize my main objection to the fundamental doctrine of each:

"1. Time is a stuff both 'resistant and substantial.' Where is the specimen on which this allegation is founded?

"2. Consciousness is to some extent independent of cerebral structure. Professor Bergson thinks he proves this by disproving a crude theory of localization of mental qualities. Will he furnish evidence of its existence apart from cerebral structure?

"3. Instinct leads us to a comprehension of life, that intellect could never give. Will Professor Bergson furnish instances of the successes of instinct in biological inquiries, where intellect has failed?

"I venture to think that, until these questions are answered, we are not called upon to consider further the merits of Professor Bergson's philosophy." EDITOR.

## KANT AND BERGSON.[1]

> "It is an incorrect and perverted usage of the word 'symbolic,' but one which is accepted by modern logicians, when it is set in opposition to the 'intuitive' mode of thought; for the symbolic is only a species of the intuitive."—Kant, *Critique of Judgment.*

We have in Kant not only the founder of criticism as a system or a method which would be appreciated for their positive qualities; but on the other hand the purely critical, or if you prefer negative, element is for the most part considered from an historical

[1] Translated from the German by Lydia G. Robinson.

point of view in its application against rationalism and against Locke and Hume. At bottom, however, Kant himself has tirelessly given expression to the propaedeutic character of his critique as among its most essential features. Therefore it seemed to him most important once for all to demonstrate metaphysics, as he found it and understood it, to be futile and impossible, in so far as it laid claim to being a system of cognitions.

Whether or not Kant had come in actual contact with historical metaphysics alone makes no difference. He undoubtedly wished to do away with metaphysics in itself. It may be objected that he opposes his criticism to that kind of metaphysics which he himself has constructed as the object of attack. Nevertheless his critique has a far broader application inasmuch as it makes metaphysics in general the object of investigation. Whoever maintains the mere possibility of a metaphysics must in some way or other decide the question which Kant himself stated and wished to have solved, namely whether metaphysics is at all possible.

In solving this problem it is a matter of indifference whether or not one employs the Kantian method of deducing the possibility of the thing sought from its postulate, from the hypothetical assumptions of the problem. Only in one way or another the critical attitude must be brought to bear upon the question. Hence the nature of metaphysics or its necessity must not be asserted and presented before its possibility is proved. Therefore it is really impossible for a Kantian to admit the methods employed by Bergson in founding a new kind of metaphysics. Nevertheless we shall first accommodate ourselves to his mode of thought so that we can not be subject to the charge of orthodox critique. Yes we will even go one step further. We will hypothetically admit that Bergson's definition of metaphysics is right. He asserts metaphysics to be the science which gets along without symbols; it is intuitive knowledge.

According to Bergson himself intuition is a sort of mental sympathy by means of which one may transfer himself into the midst of an object. Bergson avails himself of still other senses in order to make this kind of cognition comprehensible to us. It is a kind of mental auscultation, an intellectual vision. My present task is to investigate whether such an intuition is possible, whether it is at all thinkable. Bergson is satisfied simply to make the assertion. But I will first show that even granting its possibility it does not accomplish what is claimed for it.

From the very beginning intuition is something more than merely a kind of cognition. It is supposed to transfer us directly into the very being of the object, but in this being is included existence. A comprehension of existence is at the same time a comprehension of the cause of existence. The play is ceaselessly repeated, one direct leap carries us across the abyss of cognition, perception and comprehension. In intuition existence itself is posited. The more intuition is built up upon being, upon existence, the more creative and the more constructive does it itself become. A second process, that of deepening, runs parallel to this development of the concept of intuition. From a comprehension of the object, from a sinking into a strange object, from a constantly greater pouring out of the subject, intuition becomes more and more an internal process; finally, in intuition the subject comes more and more to comprehend itself, its creative nature, its most profound existence.

The stages of this development are well known. Scholasticism saw in intuition the cognition of existence or non-existence. "Intuitive knowledge of a thing is knowledge by means of which it may be known whether a thing is or is not."[2] The logical antithesis of existence and non-existence indicates even beyond that the creative cause of existence. It is only necessary for the factor of necessity to be abstracted from its logical wrappings to make it clear that a decision about existence or non-existence ultimately depends on the foundation, the positing of existence. Existence once posited, the cognition of its necessity is at the same time admitted.

Spinoza goes even one step farther:

"This kind of knowledge, i. e., intuitive knowledge, proceeds from an adequate idea of the absolute essence of certain attributes of God to the adequate knowledge of the essence of things."[3] By reference to God, existence is therefore established more securely so that the *scientia intuitiva*[4] latterly comes to include existence. Intuitive knowledge as knowledge under the form of eternity comprises this, that the essentialities of things follow from the eternal nature of God by eternal necessity.[5] And if we must remove the

[2] "Notitia intuitiva rei est talis notitia, virtute cuius potest sciri, utrum res sit vel non sit."—William of Occam, in *l. sent, prooem.*

[3] "Atque hoc cognoscendi genus (*sc. scientia intuitiva*) procedit ab adaequata idea essentiae, formalis quorundarum Dei attributorum ad adaequatam cognitionem essentiae rerum."—*Ethices*, II, Propos. XL, Schol. 2.

[4] Or *cognitio intuitiva, Eth.* V, Prop. 36 Schol.

[5] *Eth.* V, Prop. 25, 27, and Dem. 32.

factor of the creative, we must nevertheless emphasize with Spinoza himself the power and the force of this third step in cognition from which the *amor intellectualis dei* arises.

The necessity of existence in the *scientia intuitiva* can not be more emphatically expressed than in the words: "Therefore to conceive things under the form of eternity is to conceive things in so far as they are conceived through the essence of God as real entities or insofar as they involve existence through the essence of of God."[6] The climax of this development of the concept of intuition (*Intuitionsbegriff*) is Kant's interpretation of the nature of intellectual intuition (*Anschauung*). According to him it is a non-sensual active "faculty" which produces its intuition directly and at the same time the objects of that intuition by its spontaneous activity. It seems that Kant saw in Plato's Ideas the objective counterpart of this intellectual intuition, for in them as intuitions *a priori* he posits the primitive cause of all things. (*"Von einem neuerdings erhobenen vornehmen Ton in d. Philosophie, Berliner Monatsschrift, Mai,* 1796.")

Kant shares with Spinoza the association of this intellectual intuition with the divine. He differs from him in that he does not admit with Spinoza that it is possible on the part of man.

I pass over entirely the concept of intuition as worked out in mysticism. With this concept the intuition of Bergson has nothing to do.

Granted that intuition is possible, what does it accomplish? It transfers one directly into the midst of objects. What of objectivity it gains it loses in subjectivity. Its climax is its coincidence with the essence of the object, and thus is emphasized as something quite distinct from it. But if it remains distinct then it must always be outside of the center of the object. This transference into a strange object is really only a purposeless example of speculative fancy, for it is absolutely inconceivable how a subject could be so changed into an object that it would take up the object into itself, make itself equivalent to it and yet remain autonomous itself. And even if this procedure were possible we would utterly reject the dualistic theory and be satisfied with the admission that in the center of the essence of an object there is such a comprehension of this center that exactly this comprehension would always be meant and finally would be so

[6] "Res igitur sub specie aeternitatis concipere est res concipere, quatenus per Dei essentiam ut entia realia concipiuntur sive quatenus per Dei essentiam involvunt existentiam."—*Eth.* V, Prop. 30, Dem.

understood again, provided that this procedure could be represented in any way.

The process of intuition can not be presented nor can it be controlled. It withdraws from every attempt at presentation or control. In secret depths there suddenly takes place an escape, a *μετάβασις εἰς κρῆμα* pursued and extended indeed with effort but in its origin and course unknown and unknowable. Means are entirely lacking to verify its necessity and validity beyond its reality.

Every intuition is isolated, yet we do not see how a methodical and systematic connection can be possible in the sum total of intuitions. Neither an ascent, an increasing deepening, a methodical thought-action, nor a well-constructed systematic connection of cognitions. But we might perhaps waive this: intuitions crowd together in one of the most important, in the intuitive attainment of intuition itself. Thus we would have an undivided apex crowning the structure of cognition. The cognitions themselves might be of another kind. But when and in whom is this intuition to take place? Can any one attain it at any time by making sufficient effort and striving to win it? If so, I should think that exactly these preliminary conditions, the knowledge of the kind of our endeavors, would greatly concern us, and intuition itself would let our endeavor fall from us void of interest like ripe fruit. There is something infinitely wearisome about intuition. At one stroke it tears away the veil from the mystery of mysteries and then all work is performed forever. And yet not for ever. It remains finally, to be sure, the possession of its acquirer who is not in a condition to communicate it to others though he can indeed arrange to put himself in possession of it, but has the possession for himself without being able to compare it or to communicate it. So from this point weighty prospects open before us. We do not exactly see how intuition could remain as a possession with its acquirer. He must ever seek to acquire it anew, for in memory exactly that disappears which makes it intuition, namely, the lack of the symbolic, an everlasting coming and going of intuitive experiences without plan or method, without connection or aim. For each one brings with it as the supreme purpose of cognition, but only as an experience, the truth as it is given, not as it is known, comprehended and perceived.

However, the deeper we descend into the inwardness of the subject which produces the intuition, the stronger is evidenced the characteristic note of the personal life, and the more distinct becomes the absolute in itself. Assuming too that we include in these depths

the real, the cosmical center of the spiritual life, then exactly this personal element, this experience, gives it a particularly independent garb. From this point it is quite unimaginable how being and experience are to be associated together. The best we can do is to assert that the Kantian problem of cognition becomes deepened and broadened but it goes no further. Intuition, too, whose legal character and validity must be comprehended or intuitively perceived, is not a datum or a reality; but it is a problem and one that has validity.

What importance for intuition has the character of truth? Since it can not be determined either categorically or by means of ideas and especially not by symbols, it can, to be sure, contain truth in itself—yes, according to its concept it must contain truth; but how and by what intrinsic necessity it contains truth can by no means be expressed without symbols. Only no one needs to know that a cognition or an experience contains truth (for this knowledge would be either accidental or problematic) but only to know by what necessity truth is bound to a cognition or an experience. Then too the mere possession of truth is worthless so long as it is not known that it has its roots in well-grounded associations.

Therefore intuition must be rejected as a postulate because it cannot serve to give any one an accidental experience of finding himself in the center of an object. The primeval dream of humanity to be able to know finality, to be able to possess everything, to penetrate into the innermost kernel of things, is in itself contradictory and untenable. Of what use is it to me to be in the center of an object if I do not have besides an intuition that this is the case, that it actually is the case? Did not Descartes remind us of the possibility that a conception could be produced in us arbitrarily and delusively from an outside cause? He comes to the conclusion that our fancy can actually transport us into a foreign object very vividly and naturally without question, and yet with an easy effort if not simultaneously we can have the consciousness that it was simply an image of our fancy. In the moment when I by means of certain efforts of the imagination live in a vividly portrayed character of romance I have nothing but this imaginative figure within me and it is utterly impossible for me to accompany this process, which likewise is reflected in the very threshold of consciousness, with a particular act of consciousness which includes it as object or even only with the mere idea of the ego. On the other hand it

is very possible to emphasize and to comprehend an act of imagination in a particular process of consciousness.

Accordingly since intuition is said to transfer one directly into an object, this is analogous to throwing a piece of sugar into water. The sugar is dissolved; "it" is indeed in the water, but the "piece" of sugar is not in it.

Intuition is a sort of absolute cognition. If intuition is possible, if we could penetrate at even one point into the mystery of the universe, the force of our cognition would be weakened forever at this point. At the most we would still have to assert and communicate the endeavor and the achievement if we could—at best the old traditionalism of the end of the eighteenth century. However, it is clear that intuition has already its results in great men, exceptional personalities, and that definite institutions or school buildings had them in charge. Then the incessant effort to attain possession of these intuitions would always be simply in order to gain the same possession. Of course Bergson himself does not intend this, but it is implied in the consequence of this wearisome intuition.

Bergson has foreseen the dangers that threaten, for instance, his concept of intuition. He constantly asserts the activity of intuition. There is no doubt that such an exceptional event as attaining the absolute is accompanied and introduced by attempts and efforts of an extremely energetic kind. But intuition itself is not for this reason active in any sense, although it is accompanied by activity. Exactly the last point, namely transference, in which the absolute and the comprehending subject almost coincide, must also stand on the lowest step of activity, otherwise the whole process of identity would be incomprehensible.

Bergson will undoubtedly accuse every critic of trying to assail his concept of "pure" intuition with symbols in an inadmissible way. But what if the gift of intuition refuses to come to us in spite of all our efforts? Then in Bergson's estimation we are indubitably lost as metaphysicians. It seems to me that the appeal to intuition would greatly resemble the appeal to the healthy human understanding which Bergson to be sure also invokes (p. 40). But Bergson must show us distinctly and precisely the ways and means that lead to intuition.

The intellectual experience (*Miterleben*) of the real mobility by which thinking is obliged constantly to reverse the work of thought, is claimed to be accomplished methodically. Or, rather,

only the reversion is properly claimed to be performed methodically. For does not Bergson see that we are again under the spell of the formulism of symbols which has just been rejected with the greatest energy? Are not mobility, reversion and method symbols just as much as cognition, validity, categories, etc.? Bergson himself sees how difficult it is for "the intuition once attained to find a mode of expression and application corresponding to our habits of thought and offering us in firmly postulated concepts the secure supports of which we are so in need."

But everything finally comes to this, that if one were constantly to imagine that he could transfer himself directly into the midst of an object by exerting a special energy, this procedure would remain epistemologically and metaphysically valueless so long as it does not succeed in establishing the scope and degree of its validity, its internal truthfulness, the origin and structure of its composition, etc. It never depends on the cognition or experience in itself, nor on its kind which may be described as much as one wishes, but always on laying the foundation of cognition on a firm interrelation. Upon what is the certainty of an intuition, and necessarily of its contents, based? In what consists the security that I actually grasp the essence of a thing, that I really am in the center of the object?

Bergson thinks he possesses one means of comparison. He is convinced that the consciousness we have of our own person in its "continuous course leads us into the interior of one reality after whose pattern we must construct the rest."

He also upsets Kant's theory of the unknowability of the ego. ...."Accordingly I have no knowledge of myself, what I am, but merely how I appear to myself."[7]

We nowhere find in Bergson any attempt formally to oppose the well-known deductions of Kant. At the same time he arms himself against Kant's proofs. He accuses Kant of "misconstruing the union of the sciences and metaphysics with intellectual intuition." It would have been more correct to say that Kant has opposed it with all his energy. Kant did not in the least accuse metaphysics of being empty speculation; he was even the first to point out the necessity of the metaphysical impulse. But he has undertaken to show that metaphysics could never stand as a system of cognitions. His problem was formulated: Is metaphysics everywhere possible as a unity of cognition? and his answer was firm

[7] *Cr. of Pure Reason*, II, p. 157, cf. 135, 399 ff., and also the alteration of the first edition. *Proleg.*, 136 ff.

and decided, Not at all. With equal energy he rejected intellectual intuition as cognition. Whoever makes both assertions repeatedly is obliged to shatter and overturn Kant's critique in its fundamentals.

But this is not the case since Natorp's and Cohen's system has been called a "dream" (p. 52). Ultimately we will have to dispose of the idea which ascribes to Kant such a dogmatism as even his own opponents have not consistently perfected, which maintains that after Kant "the main task of criticism is to determine what the intellect is supposed to be and what the object" (p. 52). Equally dogmatic is the postulate that is ascribed to Kant that the intellect is incapable of doing anything but "Platonize, i. e., cast every possible experience into previously existing moulds" (p. 53).

To be sure we are no longer satisfied with the conception of metaphysics as it appeared to Kant. Likewise is it far indeed from us absolutely to deny its possibility as he did. We maintain that greater depths of the soul, which Kant also divined (synthetic oneness of apperception) can become present to us, but not by the help of intuition, of intellectual perception (*Anschauung*), but in an energetic apprehension, in an active realization of its infinite content. Hence we consciously abandon cognition and its ways and means which Bergson desires to broaden and deepen anew. For by means of intellectual perception we fall again and still deeper into the miserable intellectualism in which we long enough have lain imprisoned.

Intuition indeed is to be divested of all intellectuality. Apart from the fact that it thus incurs the loss of all power of cognition, it becomes in addition a kind of assimilation of the object which repeats in some way or other its content, and is everything else, except cognition or comprehension. And yet finally the resultant, the sum total of the intuitive performance must be analogous to "experience." The bare object must be distinguished from the object in the confusion of intuition. And right here lies the problem. For that an object can be concerned with intuition would be possible in itself. But who could undertake to find out by any other means than through intuition what the characteristic feature of the object is, and on the other hand the content of the perfected intuition?

Assuming the possibility of intuition, it does not accomplish what is claimed for it. The leap into the thing buries the one who takes it. Intuition assumes a thing which outside of and independent of itself does not exist. Intuition is not only unfruitful, it is even impossible.

For this statement I hope to bring forward convincing proofs. All the varied results and evolutions of modern epistemology possess the common feature of interpreting cognition as complete and immanent. It deduces all single factors and elements from the problems and laws of cognition itself but does not construct them *a priori* upon metaphysical foundations. For although the constitutive features of the nature of cognition might be based on metaphysical relations yet that which makes cognition cognition can be ascertained only by their surrender. Hence a kind of cognition which assumes the "thing" as given according to its existence and its nature is self-contradictory. Cognition exactly implies that it gains, attains, performs something. A mere transmigration into the center would either signify a mere presence in the thing or a replacement of the objective central point by an assimilating subject. In either case no decision is reached about cognition itself.

The tendency of modern epistemology is to look upon everything as under the law of cognition. Bergson tries to push the thing, the "inwardness" (*Innere*) of cognition, before it and place it outside. Moreover the "being in the center" is the characteristic feature of cognition. But while Bergson stops here the modern epistemologists begin to lay their foundation at just this point. The method by which the center of the object is reached is most important. That cognition reaches this point is implied in its concept and need not be so greatly emphasized. But how it attains it is important, and it makes the matter rather easy if the proper cognitive process in the mysterious leap into the center is allowed to plunge undiscerned. The problem is not how one can be "in" a thing, but how in this center he can be active, and of what kind is the assimilation or establishment of the center.

Then too the idea of a "central point" is an uncertain one because it makes the end disappear and yet holds fast to the goal even though undetermined. Thus the methodical character of cognition is entirely overlooked, and its infinite exertion does not come to its own.

The interrelated cosmos of the objects of cognition is knocked into nothing, and is firmly bound to unchangeable points. Intuition wills everything and is itself nothing.

However greatly much in Bergson's work appeals to us, especially the significance of the real as something moveable (although the last word does not seem to have been spoken even here), yet

we must take issue as energetically against the theory of intuition as against his pragmatism (page 54).

I have not formulated the above considerations systematically but have rather adopted the rhetorical style of the French in order to remain as objective as possible. It seems to me the time has not yet come for a far-reaching reflective critique, since Bergson has promised a more conclusive argument for his theory in the future. In any case he must without question come to an understanding with Kant; for to uphold metaphysics according to Kant is difficult, but to introduce intuition again is by far the most difficult.

DR. BRUNO JORDAN.

EINBECK, GERMANY.

## MAUPERTUIS AND THE PRINCIPLE OF LEAST ACTION.

The present investigations are concerned with the history of the Principle of Least Action in the hands of Maupertuis, Euler and others. The subject is of great importance in the history of mechanics, both because the principle of least action became, in the hands of Lagrance, "the mother," as Jacobi expressed it, "of our analytical mechanics," and because the animistic tendency displayed in the search for a maximum or a minimum principle in physics undoubtedly had a great influence on such moulders of mechanical theory as Euler, Lagrange (in his early work).[1] Hamilton, Gauss, and, in

[1] Besides Lagrange's early printed works, his correspondence with Euler allows us to form some impression of the stimulating effect which the principle of least action had on Lagrange's mind at the beginning of his career. Lagrange's correspondence with Euler extends from 1754 (probably: the year is not given) to 1775 and is reproduced in the *Œuvres de Lagrange*, vol. xiv, pp. 133-245. Already in 1754 Lagrange announces (*ibid.*, p. 138) that he has made "some observations about the maxima and minima which are in the actions of nature." In a letter of August 12, 1755 (*ibid.*, pp. 138-139) Lagrange informs Euler that he had a new and simpler method of solving isoperimetrical problems and (*ibid.*, pp. 140-144) gives a full statement of it (cf. Euler's reply, *ibid.*, pp. 144-146). This discovery of what was afterwards called "the calculus of variations" certainly gave the principle of least action an additional attractiveness to Lagrange; he speaks, in a letter of May 19, 1756, of his meditations "on the application of the principle of least action to the whole of dynamics" (*ibid.*, p. 155; cf. pp. 156, 158, 161, and the final sentences of Lagrange's first printed paper in the first volume of his *Œuvres*). Lagrange's interest in the principle of least action seems to have evaporated when he observed that, when developed, the integrand is the variational form of d'Alembert's principle, and that it is simpler and equally effective to start with the equations of motion divorced from the integration. This is Lagrange's point of view in 1788. The earliest date at which this change in point of view is shown is, so far as I can find, 1764 (early memoir on the libration of the moon). In a letter of Sept. 15, 1782, to Laplace, Lagrange

our own times, Willard Gibbs. I have avoided, as much as possible, entering into merely biographical details and details of the great controversy between Maupertuis, König, Euler, and Voltaire about this very principle, in so far as they have no value in the history of science. But I have been very careful to give accurate and detailed references to the books and memoirs where everything relevant, so far as I know, may be found. I mention this expressly, because much in this chapter of the evolution of mechanics—one may even say, of thought in general—has been misquoted or misunderstood by even eminent authorities. Unless the contrary is stated, all the books referred to have been consulted either by my assistant, Miss Harwood, or by myself.[2]

I.

Pierre Louis Moreau de Maupertuis[3] was born at Saint-Malo in 1698 and died at Basel in 1759. He was the first French Newtonian;[4] was the author of several papers on the figure of the earth and the leader of that well-known French expedition which measured an arc of the meridian in Lapland, confirming the deduction from the Newtonian theory that the earth is flatter at the poles;[5]

says (*Œuvres*, vol. xiv., p. 116) that he has almost finished a mechanical treatise uniquely founded on "the principle or formula" given in section 1 of his memoir of 1780 on the libration of the moon.

[2] Adolf Mayer (*Geschichte des Princips der kleinsten Action.* Akademische Antrittsvorlesung, Leipsic, 1877, p. 7) reports that among the manuscripts left by Jacobi are fragments of a history of the principle of least action of which he has made use.

[3] There is a biography of Maupertuis by La Beaumelle (*Vie de Maupertuis par L. Angliviel de la Beaumelle; ouvrage posthume, suivi de lettres inédites de Frédéric le Grand et de Maupertuis, avec des notes et un appendice*, Paris, 1856). Cf. also Samuel Formey, *Eloge de M. de Maupertuis* (read in 1760), reprinted, with additions and corrections by de la Condamine and Trublet, in 1766 in the *Histoire de l'Académie de Berlin* for 1759, pp. 464-512; and Emil du Bois-Reymond, *Maupertuis; Rede....*, Leipsic, 1893 (on La Beaumelle's book, see pp. 72-81).

[4] La Beaumelle, *op. cit.*, p. 16; du Bois-Reymond, *op. cit.*, pp. 17-18. See Maupertuis's papers in the Paris *Mémoires* for 1732-1736; and *Discours sur les différentes figures des astres, avec une exposition des systèmes de MM. Descartes et Newton*, published anonymously at Paris in 1732 and again in 1742 (not seen), and the popular part of it is most conveniently consulted in the *Œuvres de Mr. de Maupertuis*, Lyons, 1756, vol. i, pp. 79-170. Cf. La Beaumelle, *op. cit.*, pp. 23-34; I. Todhunter, *A History of the Mathematical Theories of Attraction and the Figure of the Earth from the Time of Newton to that of Laplace*, London, 1873, vol. i, pp. 63-76, 93-102 (this also contains an account of those works which come into the scope of the next note).

[5] La Beaumelle, *op. cit.*, pp. 34-64, 71-75, 457-458, 461-462, 467; Du Bois-Reymond, *op. cit.*, pp. 18-35; and a German translation with notes by myself, of Clairaut's book of 1743 on the figure of the earth, which is soon to appear in *Ostwald's Klassiker.*

and was Frederick the Great's President of the Berlin Academy[6] (from 1746). With Maupertuis's geometrical works we are not concerned here,[7] nor are we with those philological and anatomical speculations of his, which were so ruthlessly and unjustly parodied by Voltaire.

According to Du Bois-Reymond,[8] Maupertuis's teleological tendencies showed themselves early in his career in speculations as to what grounds the Creator could have had for preferring the law of the inverse square to all other possible laws of attraction.

Some words about Maupertuis's personal character are necessary. When Maupertuis returned from Lapland, there was great opposition in some quarters to the reception of his results. This foolish opposition had a bad influence on Maupertuis: his never small feeling of self-importance increased, and he became embittered.[9] On the other hand, he was given, as President of the Berlin Academy, almost unlimited powers, even as regards the payment of the members' pensions,[10] and this may partly explain, as Carlyle suggests in his *Frederick the Great,* the tiring chorus of praise that breaks out in the Berlin *Histoire* whenever any of the members have occasion to mention Maupertuis's name. In the course of our discussions, too, we shall have, in order to correct a strange error about Maupertuis and the principle of least action made by Lord Morley in his well-known book on *Diderot and the Encyclopœdists,* to touch upon the question as to whether Maupertuis was a materialist or not.[11]

II.

Maupertuis read to the Paris Academy on the 20th of February, 1740, a memoir entitled: "Loi du Repos des Corps."[12] He began by remarking that demonstrations *a priori* of such principles as that

[6] La Beaumelle, *op. cit.,* pp. 65-68, 76, 91-98, 104; du Bois-Reymond, *op. cit.,* pp. 36, 38, 39-42.

[7] La Beaumelle, *op. cit.,* pp. 15-16. 18-19, 22-23, 460-461; du Bois-Reymond, *op. cit.,* p. 16; M. Cantor, *Vorlesungen über Geschichte der Mathematik,* vol. iii, 2d ed., Leipsic, 1901, pp. 774-775, 786.

[8] *Op. cit.,* p. 18. The place where this speculation is given is in the *Figure des Astres* (*Œuvres,* 1756, vol. i, pp. 166-170).

[9] Du Bois-Reymond, *op. cit.,* p. 33.

[10] *Ibid.,* p. 40; La Beaumelle, *op. cit.,* p. 107.

[11] In the course of this article, we shall refer to Mach's work on mechanics as *Mechanik* and *Mechanics,* as we have done before (*Monist,* April, 1912).

[12] *Histoire de l'Académie royale des sciences. Année 1740. Avec les Mémoires de Math. et de Phys. pour la même Année,* Paris, 1742, pp. 170-176; *Œuvres,* 1756, vol. iv, pp. 45-63.

of the conservation of *vis viva* "cannot apparently be given by physics; they seem to belong to some higher science."

Maupertuis sought for a general law in statics analogous to the known theorem that, in any system of elastic bodies in motion, which act upon one another, $\Sigma m.v^2$ is constant, and found that: In order that a system of bodies of which each is attracted to a center by a force varying as the $n$th power of the distance from that center, should remain in equilibrium, it is necessary that

$$\Sigma m.f.z^{n+1},$$

where $f$ is the intensity of the force which acts on $m$, and $z$ is the distance of the mass $m$ from its center of force, is a maximum or a minimum. In the proof, by showing the truth of the principle in two classes of cases, he concludes that as, for equilibrium

$$\Sigma m.f.z^n.dz = 0,$$

the above sum must be a maximum or a minimum.[13]

In an "Addition" added to the reprint in the *Œuvres*,[14] Maupertuis remarked that his law holds if the forces are proportional to functions Z of the distances $z$, and then the law is that

$$\Sigma m.f.\int Z.dz$$

must be a minimum.[15]

III.

Maupertuis's first enunciation of the law of the least quantity of action was in a memoir read to the French Academy on April 15th, 1744, entitled: "Accord de différentes Loix de la Nature qui avoient jusqu'ici paru incompatibles."[16] The laws in question ap-

[13] If there is one constant force on all the masses, and its center is at an infinite distance from the system, the center of gravity of the system must be as far as possible from, or as near as possible to, this center, for equilibrium to subsist.

[14] Vol. iv, pp. 62-63. It should be remarked that Euler, in a paper quoted below in the Berlin *Histoire* for 1751, pp. 171-173, had pointed out: (1) that it is not necessary that the forces are proportional to like powers of the distances, provided that we do not neglect the coefficients $1/(n+1)$ when they are different for the different bodies on which the forces act (p. 171); (2) that the forces need not be supposed to be proportional to functions (*fonctions quelconques*) of the distances, and if the force is V instead of $fz^n$, $\Sigma f.m.V.dz$ will then be a maximum or a minimum—according to the kind of equilibrium (p. 172); and (3) that the whole distance of each body from the centers of forces need not be considered, but, if convenience of calculation requires it, we need only consider the distances of the bodies from fixed points on the lines of direction of the forces (pp. 172-173).

[15] Maupertuis does not add: "or a maximum." The subject of this memoir of 1740 and its connection with the principle of least action were afterwards greatly developed by Euler. Cf. also Mach, *Mechanik*, pp. 69-75; *Mechanics*, pp 68-73.

[16] *Histoire de l'Académie*; *Année* 1744 (Paris, 1748), pp. 417-426; *Œuvres*, 1756, vol. iv, pp. 3-18 (with the addition referred to below).

pear[17] to be those of the reflection and of the refraction of light. When a ray of light in a uniform medium travels from one point to another, either without meeting an obstacle or with meeting a reflecting surface, nature leads it by the shortest path and in the shortest time. But when a ray is refracted by passing from a uniform medium to one of different density, the ray neither describes the shortest space nor does it take the shortest time about it. As Fermat showed, the time would be the shortest if light moved more quickly in rarer media, but Newton proved that, as Descartes had believed, light moves more quickly in denser media. Maupertuis's discovery was that light neither takes always the shortest path nor always that path which it describes in the shortest time, but "*that for which the quantity of action is the least.*"

"I must now explain," he went one,[18] "what I mean by *the quantity of action.* A certain action is necessary for the carrying of a body from one point to another: this action depends on the velocity which the body has and the space which it describes; but it is neither the velocity nor the space taken separately. The quantity of action varies directly as the velocity and the length of path described; it is proportional to the sum of the spaces, each being multiplied by the velocity with which the body describes it. It is this quantity of action which is here the true expense (*dépense*) of nature, and which she economizes as much as possible in the motion of light."

Then Maupertuis found, as a consequence of his principle, that the sine of the angle of incidence is to the sine of the angle of refraction in the inverse ratio of the velocity of the light in each medium.[19] After showing that the law of reflection also follows from

[17] Maupertuis afterwards stated (see below, section V) that the agreement was between the laws of the motion of light and mechanical laws. I have given below my grounds for almost suspecting that this was not what Maupertuis originally meant.

[18] *Histoire de l'Académie,* 1744, p. 423; *Œuvres,* vol. iv, p. 17. Notice that *here,* in the *general* definition, *mass* is not mentioned. This is another reason for believing that, at first, Maupertuis only considered the motion of light-corpuscles, and not that of ordinary matter.

[19] Cf. Mach, *Mechanik,* pp. 397-398; *Mechanics,* pp. 367-368. Using Maupertuis's and Mach's figure, CRD is the horizontal refracting plane, AR is the incident and RB the refracted ray (A and B being any points chosen on these respective rays), $m$ the velocity of light along AR and $n$ the velocity along RB. Then Maupertuis says correctly that, according to his principle, $m$.AR $+n$.RB must be a minimum. That is to say

$$d[m\sqrt{(AC^2+CR^2)} + n\sqrt{(BD^2+DR^2)}] = 0,$$

whence, carrying out the differentiations, observing that AC and BD are constant, and $d(CR) = -d(DR)$,

$$(CR/AR : DR/BR) :: n : m, \text{ or } (\sin CAR/\sin RBD) = (n/m),$$

which is correct on the corpuscular hypothesis; Mach's criticism that the

his principle of the least quantity of action, Maupertuis concluded:[20] "We cannot doubt that all things are regulated by a supreme Being, who, while he has imprinted on matter forces which show his power, has destined it to execute effects which mark his wisdom;....." And:[21] "Let us calculate the motion of bodies, but let us also consult the designs of the Intelligence which makes them move."

It is of interest, in connection with the dispute with König which arose afterwards, to read the note which Maupertuis appended to the reprint in his *Œuvres*:[22]

"When I read the preceding memoir in the Paris Academy of Sciences, I only knew of what Leibniz had done on this matter by what M. de Mayran says of it in his memoir on the reflection of bodies in the Paris *Mémoires* for 1723. Like him, I had confused this opinion of Leibniz's with that of Fermat...."

Then he gave,[23] after Euler,[24] the full opinion of Leibniz.[25]

Now we shall see below that Maupertuis in the *Histoire* for 1752 said that he had "adopted" Leibniz's definition of *action*. We have no means of knowing how far, if at all, Maupertuis was indebted to the ideas of Leibniz.

IV.

There is nothing on the subject of the principle of the least quantity of action in the *Histoire de l'Académie de Berlin* (which contains the *Mémoires* of the various classes of the Academy) for 1745; but, in the *Histoire* for 1746, published in 1748, Maupertuis

reciprocal values appear instead of the actual ones is only true, as P. Stäckel observed in the *Encykl. der math. Wiss.*, vol. iv, part i, 1908, p. 491, on the undulatory theory, which Maupertuis, as a good Newtonian, did not adopt.

Further, Maupertuis's principle *does* state that $m.AR+n.RB$ (which is what $\int v.ds$ reduces to here) is to be a minimum. This was contested by Mach (but cf. *Mechanik*, p. 406; *Mechanics*, pp. 375-376).

Du Bois-Reymond (*op. cit*)., pp. 48-49) speaks of the example of the motion of light which Maupertuis chose in 1744 to illustrate his principle being "not happily chosen," because experiments have proved that the velocity of light in air is greater than that in water—the opposite state of things to that which the emission theory required.

[20] *Œuvres*, vol. iv, p. 21.

[21] *Ibid.*, p. 22.

[22] *Ibid.*, p. 23.

[23] *Ibid.*, pp. 23-28. In the text of the memoir of 1744, Maupertuis (*ibid.*, p. 15) thus mentioned Leibniz: "Leibniz wished to conciliate the opinion of Descartes [that light moves more quickly in the denser media] with final causes; but he did this only by suppositions which could not be sustained, and which did not square with the other phenomena of nature."

[24] *Hist. de l'Acad. de Berlin*, vol. vii, 1751, pp. 205-209.

[25] *Acta Eruditorum*, 1682 (not seen).

has[26] a memoir: "Les Loix du Mouvement et du Repos, déduites d'un Principe Métaphysique."

This memoir begins with the prefatory remark:[27] "I gave the principle on which the following work is founded on April 15th, 1744, in the public assembly of the Royal Academy of Sciences of Paris, as the *Acta* of this Academy testify." Then Maupertuis refers to Euler's *Methodus inveniendi* of 1744,[28] and the special pleasure that the Appendix gave him, "as," he says, rather patronizingly and in words which led some[29] to suppose that Euler merely applied Maupertuis's principle, "it is a beautiful application of my principle to the motion of the planets, of which this principle is in fact the rule."

The memoir is composed of three parts: (1) Examination of the proofs of the existence of God, which are drawn from the wonders of nature;[30] (2) The thesis that these proofs must be sought in the general laws of motion, and that the laws according to which motion is conserved, distributed, and destroyed are founded on the attributes of a supreme intelligence;[31] and (3) Investigation of the laws of motion and rest.[32] In the third part, Maupertuis[33] states the general principle that "when some change happens in nature, the quantity of action necessary for this change is the smallest possible," and adds: "*The quantity of action* is the product of the mass of the bodies by their velocity and by the space which they describe. When a body is transported from one place to another, the action is greater in proportion as the mass is greater, as the velocity is greater, and as the path by which it is transported is longer." From this principle, Maupertuis deduces the laws of impact of hard (or inelastic) and elastic bodies,[34] and of the lever.[35]

[26] Pp. 267-294. The mathematical (third) part of this memoir is, in part, identical with "Recherche des Loix du Mouvement" in the *Œuvres*, vol. iv, pp. 31-42; the theological part is included in the *Essai de Cosmologie* to which we will soon refer.

[27] *Histoire de l'Acad. de Berlin*, 1746, p. 267. This note was repeated in Maupertuis's *Œuvres*, vol. i (see below).

[28] See below, section IX.

[29] For example La Beaumelle, *op. cit.*, p. 85.

[30] *Histoire de l'Acad. de Berlin*, 1746, pp. 268-277.

[31] *Ibid.*, pp. 277-287.

[32] *Ibid.*, pp. 287-294.

[33] *Ibid.*, p. 290; *Œuvres*, vol. iv, p. 36.

[34] *Histoire*, pp. 290-293; *Œuvres*, vol. iv, pp. 36-42.

[35] *Histoire*, p. 294; not in the *Œuvres*. The explanation of this omission given by Maupertuis (*Œuvres*, vol. i, p. xxvii) is that this problem is too limited (as the directions of the forces of weight are all supposed to be parallel to one another and at right angles to the straight lever); but the "Loi du

When treating of impact of hard (inelastic) bodies of masses A and B, which move with the velocities $a$ and $b$ respectively in a straight line and in the same sense, Maupertuis considers the spaces ($a$ and $b$) *described in a certain time* (the unit of time), so that $m.v.s$ becomes $m.v^2$, as Mach notices, and so he points out Maupertuis's inconsistency.[36]

Let A move faster than B, so that A catches B up and infringes on it, and let the common velocity of A and B after the impact be $x$ (less than $a$ and greater than $b$). "The alteration which has happened in the universe consists in that the body A which moved with the velocity $a$ and which in a certain time described a space equal to $a$ only moves with the velocity $a$ and describes a space equal to $x$, while the body B which only moved with the velocity $b$ and described a space equal to $b$ moves with a velocity $x$ and describes a space equal to $x$. This change is, then, the same as would have happened if, while A moved with the velocity $a$ and described a space equal to $a$, it had been carried backwards through a space equal to $a-x$ on an immaterial plane moving with the velocity $a-x$, and while B moved with the velocity $b$ and described a space equal to $b$, it had been carried forward through a space equal to $x-b$ on an immaterial plane moving with a velocity $x-b$. Now, whether A and B move with their own velocities on movable planes or they are at rest there, as the movement of these planes charged with bodies is the same, the quantities of action produced in nature will be $A(a-x)^2$ and $B(x-b)^2$, and their sum must be as small as possible." This gives

$$2.A.a.dx+2.A.x.dx+2.B.x.dx-2B.b.dx=0,$$

whence

$$x=(Aa+Bb)/(A+B).$$

In this case, where the bodies move in the same direction, the quantity of motion destroyed and the quantity produced are equal, and the total quantity of motion remains, after the impact, the same as it was before. If the bodies move towards one another it is easy to apply the same reasoning; or it is sufficient to consider $b$ as negative with respect to $a$. Then the common velocity will be

$$x=(Aa-Bb)/(A+B).$$

If A and B are perfectly elastic, and move in the same direction with velocities as before, except that $\alpha$ and $\beta$ are the respective

repos" of 1740, given in vol. iv of the *Œuvres*, is a *general* principle of statics and "agrees so perfectly with the principle of the least quantity of action that we may say that it is only the same thing."

[36] *Mechanik*, pp. 395-396, 398; *Mechanics*, pp. 365-366, 368.

velocities after impact, "the sum or the difference of these velocities after the impact being the same as it was before," then, by analogous considerations on the change which has happened in nature, Maupertuis arrives at the conclusion that the quantity of action is here

$$A(a-\alpha)^2+B(b-\beta)^2$$

and this, when minimized, since

$$\beta-\alpha=a-b \text{ and thus } d\beta=d\alpha,$$

gives

$$\alpha=(Aa-Ba-2Bb)/(A+B), \qquad \beta=(2Aa+Ab-Bb)/(A+B).$$

Here the sum of the *vires vivae* is conserved on impact, but this is not the case with hard (inelastic) bodies.

To find the law of the lever Maupertuis considers masses A and B attached to the ends of an immaterial lever of length $c$, and seeks the point, at a distance $z$ from A, around which they are in equilibrium. For this purpose he seeks the point around which, if the lever receives some small movement, the quantity of action is the smallest possible. Then A and B, on this movement being imparted to them, describe small arcs similar to one another and proportional to the distances of these bodies from the point sought. These arcs will be the spaces described by the bodies and at the same time will represent their velocities. Thus the quantity of action will be proportional to

$$Az^2+B(c-z)^2$$

and this, when minimized, gives

$$z=Bc/A+B.$$

v.

In the "Avertissement" to the fourth volume of his *Œuvres,* Maupertuis says of the memoir of 1744: "I show the agreement of the laws which light follows in its reflection and its refraction with those which all other bodies follow in their motion." In point of fact, this is not quite the case: he shows how *both* the law of reflection and that of refraction could, on the corpuscular hypothesis, be deduced from one principle; but, in the whole memoir, other motions than that of light were only referred to shortly. The law that, in a uniform medium, light moves in a straight line is common, he says,[37] to all bodies: they move in a straight line unless some external force deflects them; and the law of reflection is the same as that

[37] *Paris Histoire,* 1744, p. 418; *Œuvres,* vol. iv, p. 7.

followed by an elastic ball impinging on an unbreakable surface. But no like explanation of the law of refraction had been given.

Later on, Maupertuis[38] adds a note to his definition of the quantity of action as $\Sigma s.v$: "As here there is only one body, we abstract from its mass."

VI.

Maupertuis's *Essai de Cosmologie* was published in 1751,[39] and consists of three parts: (1) Examination of the proofs of the existence of God, which are drawn from the wonders of nature; (2) Deduction of the laws of motion from the attributes of the supreme intelligence; and (3) Spectacle of the universe. No part of the work is stated mathematically, and the third part is a rhetorical sketch of the solar system, in which the principle of the least quantity of action is not mentioned.[40] The two first parts are practically the two first parts of the memoir of 1746.

[38] *Œuvres*, vol. iv, p. 17. This note is not in the original memoir of 1744 (the paragraph in the text to which the note refers is on p. 423 of this memoir), but was first added, as a marginal note, in the *Essai de Cosmologie* of 1751. These facts suggest that the mechanical applications of Maupertuis's principle were, at least, not clear to Maupertuis in 1744. For my own part, I cannot help almost having the impression from a study of the original memoir of 1744 and its reproduction, with comments, in the *Œuvres* of 1756, that the laws of nature referred to in 1744 are the laws of catoptrics and dioptrics, whereas afterwards Maupertuis, because of the discovery communicated in his memoir of 1746, tried to persuade possibly himself and certainly his readers that the laws were more general laws of nature. Cf. Note 18, Section III, above.

Formey, in the *Eloge* quoted at the beginning of this paper, says (p. 496): "Il y [in the memoir of 1744] étoit principalement question des loix qui suit la lumière, surtout lorsqu'elle passe d'un milieu diaphane dans un autre."

[39] *Essay de Cosmologie*. Par M. de Maupertuis, Leyden, 1751. At the end (pp. 81-104) is a reprint of the 1744 paper with the mathematics (the note referred to in section V, last note, is put in the margin of pp. 97-98); and on pp. 63-80 is a "Recherche mathématique des Loix du Mouvement et du Repos," from, says Maupertuis, the Berlin *Mémoires* for 1747 (a misprint for 1746). The *Essai* was partly reprinted in the first volume of the *Œuvres de Mr. de Maupertuis* (Nouvelle édition, corrigée et augmentée, Lyons, 1756, pp. 3-78, and the mathematical part, which was omitted in the previous editions of Maupertuis's *Œuvres*, is included in vol. iv, pp. 18-19, 36-42. On pp. iii-xxviii, is an "avant-propos" giving, among other things, an account of the Koenig incident of 1751 and its consequences. On pp. xiv-xv is the same notice about his own and Euler's works of 1744 that is at the head of Maupertuis's paper in the Berlin *Mémoires* for 1746. On d'Arcy's objections (see section XV) Maupertuis (*Œuvres*, vol. i, p. xxvi) said that "As the only objection which appears to have some foundation rests on the fact that, in the impact of elastic bodies, he has confused the change which happens to the velocities (which is real) with the change of the quantity of action (which is zero), I will make no other reply than the few words I have said about it in the *Mémoires* of our [Berlin] Academy for the year 1752" (see section XVI).

[40] However, in the second part (*Œuvres*, vol. i, p. 45), we read: "What a satisfaction for the human mind to find in the laws which are the principle

Maupertuis had a low opinion of the proofs of the existence of God from the construction of animals. Thus, somebody[41] found evidence for this existence in the folds of the skin of a rhinoceros—the animal could not move without these folds. Maupertuis[42] rather appositely asked: "What would be said of a man who should deny a Providence because the shell of a tortoise has neither folds nor joints?" And:[43] "It is not in the little details, in those parts of the universe of whose relations are known too little, that we must look for the supreme Being, but in phenomena whose universality suffers no exception and whose simplicity lays them quite open to our sight."

VII.

The reason why Maupertuis laid stress on the deduction from the principle of the least quantity of action of the laws of the impact of inelastic masses was that the law of the conservation of *vis viva* fails in this case.[44] Leibniz[45] recognized Descartes's error in thinking that, in nature, the sum of the products of the masses into their respective velocities is constant, and substituted in it the squares of the velocities for the velocities, so that the sum is what is called the *vis viva* of the system considered. But, in impact, the *vis viva* is only conserved if the bodies are elastic; and, according to Maupertuis:[46] "When we make this objection to the Leibnizians, they prefer to say that there are no hard (*durs*, inelastic) bodies in nature than to abandon their principle. This were to be reduced to the strangest paradox to which love of a system could reduce one: for what can the primitive elementary bodies be but hard bodies?"

In vain, then, said Maupertuis,[47] did Descartes and Leibniz, in

of motion of all the bodies of the universe the proof of the existence of the governor of it!"

[41] *Phil. Trans.*, No. 470. [The paper referred to is entitled: "A Letter from Dr. Parsons to Martin Folkes, Esq., President of the Roy. Soc., containing the Natural History of the Rhinoceros," and is printed in the *Phil. Trans.* for 1743, pp. 523-541].

[42] *Œuvres*, vol. i, p. 12.

[43] *Ibid.*, p. 21.

[44] *Œuvres*, vol. i, pp. xvi-xvii, 44.

[45] On Leibniz's mechanics (the conservation of *vis viva*, and so on), cf. Bertrand Russell, *A Critical Exposition of the Philosophy of Leibniz, with an Appendix of leading Passages*, Cambridge, 1900, pp. 77-99, 226-238; esp. pp. 89-90. The concept of *action* with Leibniz was not mentioned by Russell; on it cf. du Bois-Reymond, *op. cit.*, pp. 48, 51, 89-90; and Helmholtz, "Zur Geschichte des Princips der kleinsten Action," *Sitzungsberichte der Berliner Akad.*, 1887, pp. 225-236, or *Wiss. Abh.*, vol. iii, pp. 249-263. Cf. also L. Couturat, *La logique de Leibniz*, 1901, pp. 229-233, 577-581.

[46] *Op. cit.*, p. xvii.

[47] *Ibid.*, p. xviii.

different ways, imagine a world which could dispense with the hand of a Creator: no quantity which can be regarded as a cause in the distribution of motion subsists unaltered. But "Action" is, so to speak, created at each instant, and always created with the greatest economy possible; and by this the universe announces its dependence on a wise and powerful author.

Maupertuis[48] said that, because he held that the conservation of *vis viva* is not the universal principle of movement, the whole sect of Leibnizians in Germany descended on him (*je vis fondre sur moi toute la secte que M. de Leybnitz a laissée en Allemagne*), and then mentioned[49] König's having attributed some of Maupertuis's and Euler's discoveries to Leibniz. Then follows[50] an account of the incident.

As a justification of the word "action," Maupertuis[51] remarked that he had found this word quite established by Leibniz and Wolff, and did not wish to change the terms.

VIII.

When speaking of Diderot's *Thoughts on the Interpretation of Nature* of 1754, John Morley,[52] now Lord Morley, said:

"Maupertuis had in 1751, under the assumed name of Baumann, an imaginary doctor of Erlangen, published a dissertation on the *Universal System of Nature,* in which he seems to have maintained that the mechanism of the universe is one and the same throughout, modifying itself, or being modified by some vital element within, in an infinity of diverse ways.[53] Leibnitz's famous idea, of making nature invariably work with the minimum of action, was seized by Maupertuis, expressed as the Law of Thrift, and made the starting point of speculations that led directly to Holbach and the *System of Nature.*[54] The *Loi d'Epargne* evidently tended to make unity

[48] *Ibid.,* p. xix.

[49] *Ibid.,* p. xx.

[50] *Ibid.,* pp. xx-xxvi, cf. section XI below.

[51] *Ibid.,* pp. xxvi-xxvii, cf. Maupertuis's paper of 1752, described below in section XVI.

[52] *Diderot and the Encyclopædists,* vol. ii, London, edition of 1905, pp. 262-263.

[53] "As to the precise drift of Maupertuis's theme, see Lange, *Gesch. d. Materialismus,* i, 413, *n.* 37. Also Rosenkranz, *Diderot's Leben,* 1866, vol.i, p. 134."

[54] "In 1765 Grimm describes the principle of Leibnitz and Maupertuis as 'gaining on us on every side'....*Corr. Lit.,* iv, 186." [Under the date of Feb. 15, 1765, Grimm (*Correspondance littéraire philosophique et critique de Grimm et de Diderot depuis 1753 jusqu'en 1790,* new ed., vol. iv, p. 186) speaks thus

of all the forces of the universe the keynote or the goal of philosophical inquiry. At this time of his life, Diderot resisted Maupertuis's theory of the unity of vital force in the universe, or perhaps we should rather say that he saw how open it was to criticism. His resistance has none of his usual air of vehement conviction. However that may be, the theory excited his interest, and fitted in with the train of meditation which his thoughts about the Encyclopaedia had already set in motion, and of which the *Pensées Philosophiques* of 1746 were the cruder prelude."

Again:[55]

"Diderot was in no sense the originator of the French materialism of the eighteenth century. He was preceded by Maupertuis, by Robinet, and by La Mettrie; and we have already seen that when he composed the *Thoughts on the Interpretation of Nature* (1754) he did not fully accept Maupertuis's materialistic thesis. Lange has shown that at a very early period in the movement the most consistent materialism was ready and developed, while such leaders of the movement as Voltaire and Diderot still leaned either on deism and scepticism."[56]

Lange's[57] work was first published in one volume: *Geschichte des Materialismus und Kritik seiner Bedeutung in der Gegenwart* at Iserlohn in 1866. In the whole book, Maupertuis is only mentioned once. On page 224[58] it is said that people debated whether the Marquis d'Argens (Jean Baptiste de Boyer) or Maupertuis or some personal enemy of Albrecht von Haller, really wrote the *Homme machine* which De la Mettrie ironically dedicated to Von Haller.[59]

The fourth part[60] is devoted to the materialism of the eighteenth century, and consists of three divisions: De la Mettrie's *Homme machine* of 1747;[61] Holbach's *Système de la Nature, ou des lois du monde physique et du monde moral* of 1770, published, according to the title-page, in London, but really at Amsterdam, under the

of the Leibniz-Maupertuis principle of thrift, immediately after speaking of the second volume of Robinet's *De la nature,* published in four volumes 1761-8.

On Holbach's *System of Nature* (1770), see Morley, *op. cit.,* pp. 155-203.

[55] Morley, *op. cit.,* pp. 272-273.

[56] *Gesch. d. Materialismus,* i, 309, 310, etc.

[57] Friedrich Albert Lange.

[58] Cf. the references below the second edition of Lange's work.

[59] Lange, *op. cit.,* p. 72.

[60] *Ibid.,* pp. 163-229.

[61] *Ibid.,* pp. 163-186.

name of Mirabaud who had been dead for ten years;[62] and the German reaction against materialism.[63]

On the other hand, Maupertuis is often spoken of in the second edition of Lange's work, published at Iserlohn in 1873 and 1875 in two volumes under the same title,[64] and it is to this edition that Morley's citations refer. We will continue this reference to Lange's book after having given some information about Maupertuis's work of 1751, which Morley mentions.

In 1751 Maupertuis published at Erlangen, under the pseudonym of "Baumann," a Latin dissertation under the title: *Dissertatio inauguralis metaphysica, de universali naturae systemata,*[65] in which

[62] *Ibid.*, pp. 186-214.

[63] *Ibid.*, pp. 214-229.

[64] There is an English translation of this edition in three volumes, by E. C. Thomas, published at London in 1877, 1880 and 1881 (*History of Materialism and Criticism of its Present Importance*). The passages in this translation parallel to those of Morley's citations are given here.

[65] Another edition, with a French translation and with neither the place nor year of publication has been given; a third, only in French and entitled: *Essai sur la formation des corps organisées* was published by l'Abbé Trublet, with a notice and conjectures about the author, at Berlin (really at Paris) in 1754; and the French version (*Système de la Nature: Essai sur la formation des corps organisées*) was published, with a preface, in Maupertuis's *Œuvres*, 1756, vol. ii, pp. 135-168 (between pp. 160 and 161 are pages numbered 145* to *160). Diderot's *Pensées sur l'interprétation de la nature* was published anonymously at Paris in 1754 with "London" as the place of printing (Cf. Karl Rosenkranz, *Diderot's Leben und Werke*, 2 vols., Leipsic, 1866, vol. i, pp. 134-146; *Œuvres complètes de Diderot*, ed. by J. Assézat, vol. ii, Paris, 1875, pp. 1-63; cf. Assézat's "Notice préliminaire," p. 3. Maupertuis's "Réponse aux objections de M. Diderot," was printed in his *Œuvres*, 1756, vol. ii, pp. 169-184 (between pp. 176 and 177 are pages numbered 161* to *176). Cf. on all this, La Beaumelle, *op. cit.*, pp. 178-181, 200-201.

On Maupertuis's theories of generation, see La Beaumelle, *op. cit.*, pp. 86-87, 98-103; du Bois-Reymond, *op. cit.*, pp. 38-39, 44-45. The *Vénus physique* of 1745 (anonymous) was republished in Maupertuis's *Œuvres*, 1756, vol. ii, pp. 1-133. The statement that Maupertuis endeavored to explain the formation of the foetus by gravitation is one of Voltaire's libels on Maupertuis. The truth seems to be that Maupertuis, in his *Vénus* and *Système de la Nature*, as well as in one of his *Letters* ("Lettre xiv, Sur la génération des animaux," *Œuvres*, 1756, vol. ii, pp. 267-282), tried to explain this formation by the different *attractions* or (in the *Système*) psychical tendencies of the different parts. The *Lettres de M. de Maupertuis* (*sur différents sujets*) were published in 1753 and again in the *Œuvres*, 1756, vol. ii, pp. 185-340, after having been grossly caricatured by Voltaire in his *Histoire du docteur Akakia et du natif de Saint Malo* (*Œuvres complètes de Voltaire*, vol. xxiv, Paris, 1892, pp. 358-376). By the way, Letters X and XI ("Sur les loix du mouvement" and "Sur ce qui s'est passé à l'occasion du principe de la moindre quantité de l'action"; *Œuvres*, 1756, vol. ii, pp. 238-242 and 243-251 respectively) refer to the principle of least action; and Letter XII (*ibid.*, pp. 252-257; "Sur l'attraction") contains a short *exposé* of Maupertuis's work in introducing Newtonianism into France.

Maupertuis does not seem, by his published writings, to have been nearly so ridiculous a person as Voltaire, for personal reasons, tried to make him appear to be. And Voltaire's sarcasms have had great influence on the ideas

a hypothesis that the parts of matter have something similar to what we call desire, aversion, and memory was advanced to explain certain physiological facts. Maupertuis chose this pseudonymous fashion of giving his thoughts to the public, partly because the work of an unknown author would be less the butt of objections, and partly in order that he should not be obliged to reply to them. But he felt it necessary to reply to Diderot's *Thoughts* because his doctrines were accused of having results contrary to religion. Then he acknowledged the work: he had soon been recognized as its author. What concerns us here is that the law of least action is not mentioned in this work of Maupertuis's. Further, the *Essai de Cosmologie* of 1751 was not published anonymously or pseudonymously. Thus there seem to be no grounds for Morley's strange error.

Lange shows that the Newtonian theory is a combination of materialism in natural science with a religious faith in the spiritual constructor of the material world-machine. "The magnificent phenomena of the seventeenth century were renewed in increased splendor, and to the age of a Pascal and Fermat succeeded with Maupertuis and D'Alembert the long series of French mathematicians of the eighteenth century, until Laplace drew the last consequences of the Newtonian cosmology in discarding even the hypothesis of a creator."[66]

Maupertuis is classed with Robinet and La Mettrie as a materialist[67] on the grounds of his Latin dissertation of 1751. The English translation of the note (37) referred to by Morley is:[68] "*Comp.* Rosenkranz, *Diderot,* i, 134 ff. The pseudonymous dissertation of Dr. Baumann (Maupertuis) I have not seen, and it may be open to some doubt, according to Diderot and Rosenkranz, whether it does really contain the materialism of Robinet—that is, the unconditional dependence of the spiritual upon the purely mechanical series of external events—or whether it inculcates Hylo-

of Maupertuis formed by succeeding generations. Thus Mach (*Mechanik,* pp. 484-485, *Mechanics,* pp. 454-455) gives, I think, Voltaire's version of some of the things dealt with by Maupertuis in a *Letter* published earlier than those just mentioned. Maupertuis's *Lettre sur les progrès des sciences* was published at Berlin in 1752 and again in his *Œuvres,* 1756, vol. ii, pp. 341-399. Here is the project of founding a town where only Latin should be spoken, in order to preserve this most universal of languages (pp. 367-368), and a plea (pp. 394-398) for "metaphysical"—or, as we would say now, psychological—experiments.

[66] Lange, *Geschichte,* 2d ed., vol. i, p. 304; *History,* vol. ii, p. 16.

[67] Lange, *Geschichte,* vol. i, p. 310; *History,* vol. ii, p. 25.

[68] Lange, *Geschichte,* vol. i, pp. 315, 412-413; *History,* vol. ii, p. 31.

zoism—that is, modifications of the natural mechanism by the spiritual content of nature according to other than purely mechanical laws."

Again:[69] "Buffon began the publication of his great work on natural history in the year 1749, with the first three volumes; but it was only in the fourth volume that he unfolded the idea of the unity of principle in the multiplicity of organisms, an idea which occurs again in Maupertuis in an anonymous work in 1751, in Diderot in the *Pensées sur l'Interprétations de la Nature*, 1754, while we find it developed with great clearness and distinctness by La Mettrie as early as the *L'Homme Plante* in 1748."

This, together with the passage referred to above, when we were speaking of the first edition, about Maupertuis being considered by some to be the author of *L'Homme Machine*,[70] completes the list of Lange's references to Maupertuis in the second edition of his book.

We must add that Maupertuis, in his writings and in his life, showed the greatest respect for religion. He was by no means a materialist and atheist,[71] and the only reason, said he, that he had for replying to Diderot's *Thoughts* on his dissertation of 1751 was that Diderot stated that the dissertation, in spite of its carefully religious tone, led to conclusions which were subversive of religion.

IX.

This seems the best place to give some account of the work of a man who will now take a prominent place in the development of Maupertuis's ideas; I mean Leonhard Euler.[72]

The modern period of the discussion of maximal and minimal problems begins with Johann Bernoulli's proposal of the problem of the brachistochrone in 1696 and the consequent rise into importance of the "isoperimetrical" problems.[73] The period 1696 to 1762 of

[69] Lange, *Geschichte*, vol. i, p. 328; *History*, vol. ii, p. 52.

[70] Lange, *Geschichte*, vol. i, p. 398; *History*, vol. ii, p. 137.

[71] Du Bois-Reymond, *op. cit.*, pp. 43-44, 49-50.

[72] On the older period of the history of such problems, see Mach, *Mechanik*, pp. 453-457; *Mechanics*, pp. 421-425. This period is—like all early periods in the history of branches of science—characterized by the fact that the maximal and minimal problems are all *isolated*. This period extends as far as Newton who in 1687 solved "the first problem of the calculus of variations," the determination of the figure of the solid of least resistance (cf. M. Cantor, *op. cit.*, p. 291).

[73] Mach, *Mechanik*, pp. 457-467; *Mechanics*, pp. 425-436. A German annotated translation of some works of Johann Bernoulli, Jakob Bernoulli, and Leonhard Euler, from 1696 to 1744, is given by P. Stäckel in No. 46 of *Ostwalds Klassiker*. Cf. also M. Cantor, *op. cit.*, pp. 237-241, 384, 446-458, 533, 846-848.

the history of such problems is distinguished by the names of Johann Bernoulli, Jakob Bernoulli, and Leonhard Euler, and extends until Lagrange, in 1762, brought all these interrelated methods under the general and abstract analytical form of the calculus of variations. It is to this period that the works of Maupertuis, Euler, and their contemporaries, with which we are concerned here, belong. The leading work published in this period was the famous *Methodus inveniendi lineas curvas maximi minimive proprietate gandentes: sive solutio problematis isoperimetrici latissimo sensu accepti* which was published at Lausanne and Geneva in 1744.[74]

Mathematicians found that various problems of mechanics might be put into isoperimetrical form. Whether their tendency to do this, which was very common at that time, was due to esthetic, theological, or technical reasons, it is hard to say. Daniel Bernoulli —a son of Johann Bernoulli—remarked that certain statical problems can be treated with greater facility by isoperimetrical methods than by the usual mechanical principles; the feeling, too, that the discovery that a problem about natural objects could be put in a maximal or minimal form had a connection with the way the Deity managed things here below in making nature act by the shortest or easiest or readiest paths, and so with what were then called "metaphysical"[75] questions, undoubtedly had an influence on others besides Maupertuis—on Euler for example. But we shall see how piety and humility led Euler, though accurate, judged by the mathematical standards of those days, very cautious, and perhaps a little unimaginative,[76] to accept and admire the bold and not always accurate mechanical generalizations which Maupertuis professed to deduce from "metaphysics." But probably the esthetic satisfaction which

[74] An annotated German translation of a great part of this book was given in No. 46 of *Ostwalds Klassiker*. However, the two appendices (on the elastic curves, and on the motion of a particle round a center of force in a non-resisting medium) with which we shall be especially concerned here were not translated with the main body of the work. But the first appendix was translated, in another connection, in No. 175 of the *Klassiker* (see below, section X). An account of Euler's book of 1744 is given in M. Cantor's *Geschichte*, vol. iii, 2d ed., Leipsic, 1901, pp. 857-867.

[75] In the eighteenth century, "metaphysics" stood for—at least among mathematicians—a branch of learning which included theology, psychology, and logic. Consider the "metaphysical experiments" advocated by Maupertuis, and the "metaphysics of the infinitesimal calculus" (L. N. M. Carnot, Lagrange, and others), which meant what we mean when we say: "the logical principles of the calculus."

[76] D'Alembert, in a letter of March 3, 1766, to Voltaire (quoted by Delambre in his "Notice" in *Œuvres de Lagrange*, vol. i, p. xxi), says of Euler: "c'est un homme peu amusant, mais un très grand géomètre."

arises from stating a problem in a maximal or minimal form influenced mathematicians the most.

However this may be, to this form come many problems of statics, such as the catenary of Johann and Jakob Bernoulli,[77] and Jakob Bernoulli's problem of the elastic curve.[78] From Daniel Bernoulli's letter to Euler and from Euler's first appendix to his book of 1744, we see with what interest Daniel Bernoulli and Euler reduced this problem in the theory of elasticity to isoperimetrical methods.

These problems were all *statical* ones; and it was Daniel Bernoulli who suggested to Euler the putting of a certain *dynamical* problem into isoperimetrical form. It must be remembered that Euler, by his papers published by the St. Petersburg Academy in 1732 and 1736,[79] had placed himself at the head of the mathematical world, in the treatment of isoperimetrical problems. We must now say some words about Daniel Bernoulli and Euler and their relations to one another.

Daniel Bernoulli[80] (1700-1782) was a son of the famous Johann Bernoulli (1667-1748) and was attached to the St. Petersburg Academy from 1725 to 1733. From 1733 to 1782 he was Professor of Anatomy and Botany, and later Experimental Physics and Speculative Philosophy too, at Basel. His mathematical works[81] are on differential equations, the theory of numbers, the theory of probability, series, and mechanics[82]—principally the theorem of *vis viva*,[83] the problem of vibrating cords,[84] and hydrodynamics.[85] Leonhard Euler[86] (1707-1783), whose name as a mathematician is too well known for it to be necessary for us to refer further to his many works, came to St. Petersburg in 1727, owing to the exertions on his behalf of Daniel Bernoulli and Hermann, but left St. Peters-

[77] Cf. Mach, *Mechanik*, pp. 75-77; *Mechanics*, pp. 74-76; *Ostwalds Klassiker*, No. 46, p. 19; M. Cantor, *op. cit.*, pp. 219-220, 228, 235, 289, 384, 455, 853.

[78] Cf. M. Cantor, *op. cit.*, pp. 220-221, and Johann Bernoulli's letter of March 7, 1739, to Euler in Fuss's *Correspondance* referred to below, vol. ii, pp. 23-25.

[79] Cf. M. Cantor, *op. cit.*, pp. 846-856.

[80] M. Cantor, *op. cit.*, pp. 89-90, 550; *Encycl. Brit.*, 9th ed., vol. iii, 1875, pp. 606-607.

[81] *Ibid.*, pp. 477-481, 610, 630-632, 634-635, 640, 642-644, 688, 693, 707, 721, 851, 900, 904-906.

[82] Cf. also Mach, *Mechanik*, pp. 43-49, 326; *Mechanics*, pp. 40-47, 293.

[83] Cf. also Mach, *Mechanik*, pp. 374-379; *Mechanics*, pp. 343, 348.

[84] Cf. also Mach, *Die Principien der Wärmelehre*, 2d ed., Leipsic, 1900, pp. 96-97.

[85] Cf. Mach, *Mechanik*, pp. 440-453; *Mechanics*, pp. 403-420.

[86] M. Cantor, *op. cit.*, pp. 549-551.

burg in 1744 to become Director of the Mathematical Class of Frederick the Great's reformed Academy of Sciences at Berlin. In 1727 Euler met Daniel Bernoulli and was stimulated by him to an investigation on geodesic lines.[87] The letters addressed by Daniel Bernoulli to Euler—those from Euler to Bernoulli are unfortunately lost— from 1726 to 1755 have been published in P. H. Fuss's *Correspondance mathématique et physique de quelques célèbres géomètres du XVIIIième siècle*.[88] From this correspondence we will now make the extracts which concern our present subject.

In a letter to Euler of January 28th, 1741, Daniel Bernoulli asked whether it was not Euler's opinion that orbits about centers of force could be deduced by an isoperimetrical method.[89] As we have said, Euler's replies are lost. In a letter of December 12, 1742, Bernoulli has some further remarks on the same subject;[90] and in a

[87] M. Cantor, *op. cit.*, p. 843.

[88] St. Petersburg, 1843, vol. ii, pp. 407-655. In these letters there is frequently mention of isoperimetrical problems, but the first mention of a *mechanical* problem treated by an isoperimetrical method is on pp. 456-457 (letter of March 7, 1739) where the elastic curve, which requires a certain integral which represents the "*potential vis viva*" to be a maximum, since Bernoulli thinks "that an elastic lamina which takes a certain curvature of itself will bend in such a way that the *vis viva* will be a minimum, since otherwise the lamina would move," is referred to (other references are given on pp. 468-469, 506-507, 512-514, 533-534, 536-537). To this apparently refers what Bernoulli (p. 534) calls an *a priori* method—a speculation which contrasts oddly with the passages quoted below which are rather anti-"metaphysical." The first occurrence of a reference to a *dynamical* problem to be treated by an isoperimetrical method is that given below.

It was Daniel Bernoulli who recommended that Bousquet of Geneva should be chosen as the printer of Euler's "masterly" (*herrlichen*) treatise on the isoperimetrical method—the *Methodus* printed in 1744 (letter of Feb. 9, 1743; *ibid.*, p. 521; cf. pp. 524-525 (see extract below), 528, 529, 533 (see extract below), 541, 550, 553, 578). In a letter of September 4, 1743, Bernoulli (*ibid.*, p. 536) says: "I regret that I could not read through your additions to the treatise on isoperimeters; but I have just (*fugitivo oculo*) glanced at them." This is important in view of Euler's account (section XII below) of the date and circumstances under which these additions were made and printed.

[89] "Von Ew. möchte vernehmen, ob Sie nicht meinen, dass man die orbitas circa centra virium könne methodo isoperimetrica, wie auch die figuram terrae pro theoria Newtoniana herausbringen" (Fuss, *Correspondance*, vol. ii, p. 468).

[90] "Man kann die principia maximorum et minimorum nicht genugsam ausforschen; die trajectoriae circa centrum virium, vel circa plura centra virium, müssen gleichfalls per methodum isoperimetricorum können solviret werden, obschon man das maximum vel minimum, quod natura affectat, nicht einsiehet. Es haben also Ew. einen grossen Nutzen dadurch geschafft, dass Sie die methodum isoperimetricorum so weit perfectionnirt haben. Meiner Meinung nach ist dieses argumentum inter omnia pure analytica utilissimum, und ist dieses ein wahres Exempel, dass vel sola propositio problematis, wenn man auch die Solution nicht hätte, saepe maxima laude digna sey" (*ibid.*, p. 513).

letter of April 23, 1743, speaks[91] with praise of Euler's great treatise on the Isoperimetrical Method, suggests the addition of a treatment of the problem of the elastic curve and others like it, and then comments on Euler's discovery that $\int v.ds$ is a minimum for central orbits, that Euler has obviously communicated to him without proof, as follows:

"The observation about trajectories that $\int v.ds$ must be a maximum or minimum appears to me very beautiful and important; but I cannot see how this principle is demonstrated. Please let me know whether the principle extends to trajectories about many centers of forces. Perhaps it is only an observation *a posteriori,* owing to a discovery you may have made that the trajectories have this property, and you may not have been able to demonstrate it *a priori.*"

In a letter of September 4, 1743, Bernoulli writes:[92]

[91] Wegen Ew. herrlichen Tractat de isoperimetricis werde ich vorläufig mit demselben reden; Sie belieben nur denselben fertig zu halten. Sie könnten das problema de elastica hac methodo invenienda und andere dergleichen noch beyfügen. Ich sehe leicht, dass man die curvaturam catenae et laminae elasticae oscillantis auch darin reduciren kann; auf den modum aber bin ich noch nicht bedacht gewesen. Die meisten curvas mechanicas wird man auch dahin reduciren können. Die Observation von den trajectoriis, dass $\int v.ds$ ein maximum oder minimum seyn müsse, dünkt mich sehr schön und von grosser Wichtigkeit; ich sehe aber die Demonstration dieses principii nicht ein. Ew. belieben mir zu melden, ob sich solches auch ad trajectorias circa plura centra virium erstrecke. Vielleicht ist es nur eine observatio a posteriori, indem Sie angemerkt haben, dass die trajectoriae diese proprietatem haben, ohne solche a priori recht demonstriren zu können" (*ibid.*, pp. 524-525).

[92] "Aus Dero Brief ersehe ich, dass ich in meiner Conjectur mich nicht betrogen, wenn ich gesagt habe, dass Dero Observation circa orbitas planetarum, in quibus $\int v.ds$ vel $\int v.v.dt$ ein minimum ist, vielleicht nur a posteriori sey gemacht worden; denn nach meinen principiis kann ich solches a priori nicht einsehen. Der Herr Clairaut schreibt, dass solches auch schon von einem Engländer sey remarquirt worden. Es scheint, dass dieses nicht sowohl ein principium, als eine proprietas sey, gleich wie es eine proprietas ist elasticae, dass sie das maximum solidum generirt. Doch hab ich nicht untersucht, ob die idea maximi solidi die elasticam in omni extensione begreife. Sie können mich dieser Mühe entheben, denn ich weiss, dass Sie alle dergleichen Untersuchungen allbereits gemacht haben. Von meinem principio a priori, dass die elastica das $\int ds/rr$ ein minimum formire, hab ich mit vieler Erkenntlichkeit ersehen, aber zugleich mit Beschämung, dass Sie in Ihrem supplemento so honorificam mentionem thun. Dieses principium gehet auch an in laminis inaequaliter elasticis, wenn man macht $\int cds/r.r$ ein minimum. Die laminae naturaliter non rectae erfordern zwar einen andern calculum, aber keine andere methodum; wenn aber die laminae proprio pondere zugleich incurvirt werden, so ist es schwer, das maximum oder minimum quod natura affectat zu determiniren. Ich muthmaasse, dass man allhier muss ad maxima maximorum recurriren, wenn zweyerley Considerationen zusammen kommen. Quaeatur brevitatis gratia curva AC, quam lamina naturaliter recta AB et uniformis proprio solo pondere incurvata accipiet: fragt sich, ob nicht curva AC talis seyn könnte, dass inter omnes ejusdem longitudinis, inter eosdemque terminos positas curvas, eandemque $\int ds/rr$ habentes, das centrum gravitatis infimum locum obtineat. Wir haben Beide diese curvam directe determinirt; fragt sich also, ob man ex hoc principio eandem curvam finden

"From your letter I see that I was not mistaken in my conjecture that your observation that $\int v.ds$ or $\int v.v.dt$ is a minimum for the orbits of the planets was perhaps only made *a posteriori*; for I cannot see this *a priori* by the light of my principles. M. Clairaut writes that this property has also been noticed by an Englishman. It appears that this is not so much a principle as a property, just as it is a property of the elastic curve to generate the maximum solid. Still I have not investigated whether the idea of the maximum solid includes that of the elastic curve in all its extension...."

And in a letter of December 25, 1743, Bernoulli writes:[93]

"I doubt whether one can ever show *a priori* that the elastic curve must generate the maximum solid; I consider this as a property which is shown by calculation and that nobody could have foreseen from first principles—as little as the identity of the isochrone and the brachystochrone. Such properties are, as it were, discovered through accident by our reason, and I consider the property observed, that in orbits $\int u.ds$ is a minimum, to be on this level. I was confirmed in this opinion by learning that you only observed this property *a posteriori* and never would have found it if you had not determined the orbit by other means."

Lastly, Bernoulli's anti-"metaphysical" tendency is still more strongly shown in a passage[94] of a letter to Euler of April 29, 1747:

"Herr Ramspeck has written to my father that you are engaged in various public metaphysical controversies. You really ought not to meddle with such matters, for from you we expect only sublime things, and it is not possible to excel in metaphysics."

Euler, we know, had a strong reverence for "metaphysics" and

würde. Der calculus aber wird ohne Zweifel weitläufig seyn, und bin ich von diesem principio nicht convincirt, so dass Ew. sich schwerlich die Mühe werden geben wollen meine Conjectur zu untersuchen. Wenn solche aber richtig wäre, würde es, wie ich glaube, leicht seyn, schier aller curvarum maxima et minima a priori anzuzeigen" (*ibid.*, pp. 533-534).

[93] "Ich zweifle ob man jemals a priori werde zeigen können, dass die elastica müsse maximum solidum generiren; ich betrachte solches als eine Proprietät, die der calculus ausweiset, und die kein Mensch ex principiis novis jemals würde haben können vorhersehen, eben so wenig als die identitatem isochronae et brachystochronae. Dergleichen proprietates sind ratione nostri gleichsam accidental, und auf diesen Fuss betrachte ich auch die observatam proprietatem orbitarum, in quibus $\int u.ds$ ein minimum macht, worin ich um so viel mehr confirmirt werde, als ich errathen, dass Sie diese proprietatem nur a posteriori observirt haben und niemals würden gefunden haben, wenn Sie nicht die orbitas aliunde determinirt hätten" (*ibid.*, p. 543).

[94] "Herr Ramspeck hat meinem Vater geschrieben, dass Sie in unterschiedenen controversiis metaphysicis publicis stehen. Sie sollten sich nicht über dergleichen Materien einlassen; denn von Ihnen erwartet man nichts als sublime Sachen, und es ist nicht möglich in jenen zu excelliren" (*ibid.*, p. 621).

consequently attached to Maupertuis's *a priori* speculations a value far above his own discovery. We shall see later that, in papers published among the *Mémoires* of the Berlin Academy, he emphasizes, as he apparently did to Daniel Bernoulli, the fact that he had only discovered the minimal condition satisfied by orbital motion in an *a posteriori* manner, as if this was rather a demerit. Nowadays we would say that Euler's great caution in, for example, insisting, in his *Methodus,* that the $v$ in

$$\int v.ds$$

is to be expressed in terms of $s$ by the principle of *vis viva,* so that his minimal principle cannot be extended to the case of motion in a resisting medium, where the principle of *vis viva* does not hold, and, in later publications, the careful enumeration of cases when testing Maupertuis's statical principle, are merits. But the following extract from the first appendix on elastic curves to the *Methodus* of 1744 proves that more general "metaphysical" ideas were by no means foreign to Euler:

"For since the plan of the universe is the most perfect possible and the work of the wisest possible creator, nothing happens which has not some maximal or minimal property, and therefore there is no doubt but that all the effects in nature can be equally well determined from final causes by the aid of the method of maxima and minima as from the efficient causes."[95]

## x.

We will now return to the publications of the Berlin Academy. The only paper concerning us here in the *Histoire* for 1747,

[95] "Cum enim Mundi universi fabrica sit perfectissima, atque a Creatore sapientissimo absoluta, nihil omnino in mundo contingit, in quo non maximi minimive ratio quaepiam eluceat; quamobrem dubium prorsus est nullum, quin omnes Mundi effectus ex causis finalibus, ope Methodi maximorum et minimorum, aeque feliciter determinari queant, atque ex ipsis causis efficientibus, *Methodus,* p. 245, and cf. section XII below. (See *Ostwalds Klassiker,* No. 175, p. 18. Cf. Mach, *Mechanik,* p. 485; *Mechanics,* p. 455. Cf. also E. Dühring, *Kritische Geschichte der allgemeinen Principien der Mechanik,* 3d ed., Leipsic, 1887, pp. 293-294, 296-299, 385-400). These reflections of Dühring's are on the effects of philosophy on mechanics and Lagrange's anti-"metaphysical" tendencies. Lagrange's own words are (*Méchanique analitique,* Paris, 1788, p. 187): "....as if vague and arbitrary denominations [such as *the least quantity of action*] made up the essential part of the laws of nature and could by some secret virtue raise simple results of the known laws of mechanics to the position of final causes"; and (p. 189): "....I regard this principle [of least action] not as a metaphysical principle but as a simple and general result of the laws of mechanics."

On the principle of least action with Fermat, Maupertuis, Euler, and Lagrange, and its effect on Gauss, cf. Dühring, *op. cit.,* pp. 100-102, 218-219, 287-302, 425-430.

published in 1749, is one in the class of speculative philosophy by Samuel Formey,[96] entitled: "Examen de la preuve qu'on tire des fins de la nature, pour établir l'existence de Dieu"; in which the author comes, by a rather different way, to the same conclusions as Maupertuis (1746).

In the *Histoire* for 1748, published in 1750, there are two papers relating to our subject by Euler.[97] The first is entitled: "Recherches sur les plus grands et plus petits qui se trouvent dans les actions des forces," and he quoted with approval Maupertuis's memoir of 1746, and remarked[98] that Maupertuis had shown that in the state of equilibrium of bodies, if some small movement were to happen to them, the quantity of action would be the least. He himself, says Euler, had discovered a similar law in the motion of bodies attracted to one or many centers of forces; in this case $\int u.ds$ expresses the quantity of action. In statics[99] this principle has been long recognized. Thus, it is easy to see that a chain suspended by its ends must take such a figure that the center of gravity of the chain is as low as possible; and thus, if $x$ is the distance of the element $ds$ from an arbitrary horizontal plane, $\int x.ds$ will be a minimum for the curve of the chain, and $\int z.ds$ is the quantity of action.[100] Many other analogous cases were, according to Euler, treated by Maupertuis; and Daniel Bernoulli remarked that the curve of an elastic lamina has a minimal property, and this view was developed by Euler in Appendix i of his *Methodus inveniendi* of 1744.[101]

There are, then, two ways of solving mechanical problems: one is the direct method, and the other is, knowing the formula which must be a maximum or a minimum, by the method of maxima and minima; the effect is determined by efficient causes and by final causes respectively. But it is often very difficult to discover the formula which must be a maximum or a minimum, and by which the quantity of action is represented; and this investigation belongs

[96] Pp. 365-384.

[97] Pp. 149-188 and 189-218.

[98] *Ibid.*, p. 150.

[99] *Ibid.*, pp. 150-151.

[100] *Ibid.*, p. 151.

[101] A convenient German translation of this Appendix, with critical and historical notes by H. Linsenbarth, was given in No. 175 of *Ostwalds Klassiker (Abhandlungen über das Gleichgewicht und die Schwingungen der ebenen elastischen Kurven von Jakob Bernoulli (1691, 1694, 1695) und Leonh. Euler (1744)*). Very interesting are Euler's (pp. 18-20) theological remarks and references to the frequency with which maximal and minimal problems appeared in the mechanical work of the Bernoullis. (Cf. section IX above.)

rather to metaphysics than to mathematics. "I believe," says Euler,[102] "that we are still very far from that degree of perfection where we are able to assign, for each effect which nature produces the quantity of action which is the smallest, and deduce it from the first principles of our knowledge; and that it will be almost impossible to arrive at it unless we discover, for a great number of different cases, the formulas which become maximal or minimal. Now, knowing the solutions with which the direct method furnishes us, it will not be difficult to find *a posteriori* formulas which express the quantity of action, and then it will not be so difficult to prove their truth by the known principles of metaphysics." With this end in view, Euler investigated several problems as to the curve formed by a flexible string in equilibrium.

Euler[103] arrived at the conclusion that the expression of the quantity of action, which, when supposed to be a minimum, gives the figure of the thread, is in perfect agreement with the *Law of Rest* published by Maupertuis in 1740.

Euler's second memoir on the principle of least action in this volume is entitled: "Réflexions sur quelques Loix générales de la Nature qui s'observent dans les Effets des Forces quelconques." He emphasizes[104] that he was only led *a posteriori* to the discovery of the minimum in the case of the equilibrium of threads, and then[105] remarks: "It is the figure which a fluid mass, all of whose particles are attracted by any forces, which was the principal object of the researches of M. de Maupertuis in order to discover the general law of rest in the Paris *Mémoires* of 1740. Thus I too will consider a fluid mass, all of whose particles are attracted to as many fixed centers as is wished by forces proportional to any functions of the distances to those centers, and I will investigate the figure of equilibrium for this mass. Then I will try to discover what will be a maximum or a minimum in this figure, in order to be in a better state to determine what must be understood by the name of the *quantity of action of the attracting forces*; and afterwards I will show by some reflections the great importance of this quantity in all researches concerning the effects produced by any forces." The expression discovered in this way was again found to agree with Maupertuis's law of 1740.

[102] *Op. cit.*, p. 152.
[103] *Ibid.*, p. 180.
[104] *Ibid.*, p. 190.
[105] *Ibid.*, p. 191; cf. p. 190.

XI.

There is nothing relating to the principle of least action, nor to mechanics (except in astronomy) in the Berlin *Histoire* for 1749 (published in 1751); but in that for 1750 (published in 1752) there is[106] an "Exposé concernant l'examen de la lettre de M. de Leibnitz, alleguée par M. le Prof. Koenig[107] dans les mois de Mars, 1751, des Actes de Leipzig,[108], à l'occasion du principe de la moindre action" by Euler,[109], with the note: "As will easily be seen by reading this memoir, it is one of those whose publication may not be delayed."

König had denied the validity of the principle in the case of equilibrium, and indicated some cases in which what, according to the principle, ought to be a minimum really reduces to nothing. But, says Euler,[110] "this objection is not of great importance, since it is sufficiently recognized in the calculus of maxima and minima that it can often happen what is a minimum vanishes entirely. But although that may be so in certain cases it by no means results that one ought to extend it to all cases of equilibrium, as always necessarily happening in that state; on the contrary, there are numberless cases in which this quantity of action is not zero but is really a minimum; and this puts beyond doubt that the aim of Nature is not the nullity of action, but its minimity." Then Euler quotes the example of the catenary, and says that the quantity of action reduces

[106] Pp. 52-64.

[107] Johann Samuel König (1712-1757); Cf. M. Cantor, *op. cit.*, pp. 599-601. König was a pupil of Johann Bernoulli's at the same time as Maupertuis. (Mayer, *op. cit.*, pp. 17-18).

[108] "De Universali Principio Aequilibrii et motus in Vi viva reperto deque nexu inter Vim vivam et Actionem utriusque Minimo" (*Nova Acta Eruditorum*, 1751, pp. 125-135, 144, 162-176). König affirms that equilibrium is a result of the *nullity* of action and *vis viva* (pp. 126, 164) that in some cases the action is a *maximum*, and this would hardly be reconcilable with Maupertuis's proof of the Creator's wisdom (pp. 126, 165); and that since *action* is *vis viva* into the time, the principle is that *vis viva* is a minimum (p. 127). König, like a thorough Leibnizian, praises the theorem of *vis viva* highly ("Censeo itaque, Theoremate Virium vivarum fundamentum universae Mechanicae contineri," p. 169), and deduces statistics from it. The extract from the letter of Leibniz's is given quite at the end (p. 176) and is: "L'Action n'est point ce que vous pensés, la considération du tems y entre; elle est comme le produit de la masse par le tems, ou du tems par la force vive. J'ai remarqué que dans les modifications des mouvemens elle devient ordinairement un Maximum, ou un Minimum. On en peut deduire plusieurs propositions de grande consequence; elle pourroit servir à determiner les courbes que decrivent les corps attirés à un ou plusieurs centres. Je voulois traiter de ces choses entr'autres dans le seconde partie de ma Dynamique, que j'ai supprimée; le mauvais accueil, que le prejugé a fait à la premiere, m'ayant degouté."

[109] As we learn from a note on p. 63 of the *Histoire* for 1750.

[110] *Ibid.*, p. 53.

to the distance of the center of gravity of the chain from the center of the earth; and[111] Daniel Bernoulli's and his own researches on elastic curves.

As regards dynamics, König quoted from a supposed letter written by Leibniz to Hermann, in which "action" was defined as Maupertuis defined it and the property of being "ordinarily a maximum or a minimum" in dynamical problems remarked. König could not produce the original nor could the original be found by officials. It is not interesting now to follow the controversy much further. König did not charge Maupertuis with plagiarism;[112] but, since the principle was considered by Maupertuis and others to be of the greatest possible importance and to reflect great credit on Maupertuis, its discoverer, the Berlin Academy, of which Maupertuis was president, took up the matter with great zeal, and concluded, like Euler's report, that, on internal and external evidences, the fragment of the letter was forged, either to injure Maupertuis or to exaggerate, by a pious fraud, the merits of Leibniz.[113] The result was an unjust expulsion of König from the Berlin Academy, and the consequent culmination of Voltaire's ill-feeling towards Maupertuis.[114]

XII.

To return to the *Histoire* for 1750. To the literature of the controversy also belongs a "Lettre de M. Euler à M. [Jean Bernard] Merian" of September 3, 1752.[115] Nowadays, the only interesting part of this letter is where Euler[116] gives some details about the publication of his *Methodus inveniendi*. The defenders of König stated that they knew the *Methodus* had been in the publisher's hands at Lau-

[111] *Ibid.*, p. 54.

[112] *Ibid.*, p. 60.

[113] *Ibid.*, p. 62.

[114] On the König incident, see La Beaumelle, *op. cit.*, pp. 139-141, 143-145, 150-167, and, on Voltaire's part in it, pp. 167 *et seq.* Further du Bois-Reymond, *op. cit.*, pp. 35-36, 47, 50-66. It is now known that the fragment of Leibniz's letter was probably genuine, and part of a letter to Varignon; Cf. *ibid.*, pp. 56-57, and the references to Gerhardt's paper in M. Cantor, *op. cit.*, p. 599.

Even in 1877, Mayer (*op. cit.*, p. 19) said that the letter was without doubt forged; but Helmholtz in 1887 (*op. cit.*) showed that its genuineness was probable.

It appears that Euler only made one separately printed contribution to the discussion on König's dissertation; it is entitled: "Dissertatio de principio minimae actionis una cum examinatione objectionum Cl. Prof. König contra hoc principium factorum," Berlin, 1783. We have not seen this work, but only quote it from the Bibliography in Fuss's *Correspondance*, vol. i, p. xciv.

[115] *Ibid.*, pp. 520-532.

[116] *Ibid.*, pp. 525-526.

sanne since 1743, a circumstance which would give Euler priority over Maupertuis. This, says Euler, is correct in so far as it concerns the treatise itself, which he had finished some years before it appeared, but he only made the additions since he had sent the manuscript to Lausanne, and only shortly before the publication of the book towards the end of 1744. Further, he had communicated this supplement to nobody before printing it.

"When," says Euler,[117] "I used the method of maxima and minima to define the trajectories which are described by bodies attracted by any central force, I do not pretend to have been beyond what MM. Bernoulli and others have done when they determined by the help of the same method the curvature of the catenary, that of a piece of linen filled with liquid, and other curves of the same kind. Such investigations only furnish particular principles which can hardly be extended further than the cases to which they are applied. On the other hand, it is a question here of a universal principle, from which all the former principles should result, and which can be regarded as a Law established in all the phenomena of nature; which would render its discussion less the part (*du ressort*) of Mathematics than of Metaphysics, on the principles of which this doctrine should be founded. Also, although for long people have not doubted that, in all natural effects, there is a maximal-minimal principle which determines them, nobody before the Illustrious President of our Academy has even suspected in what elements this principle was contained and how it could be accommodated to all cases.[118] As

[117] *Ibid.*, pp. 526-527.

[118] Cf. *Methodus*, pp. 309, 320. The actual quotations are: (1) "Quoniam omnes naturae effectus sequuntur quandam maximi minimive legem; dubium est nullum, quin in lineis curvis, quas corpora projecta, si a viribus quibuscunque sollicitentur, describunt, quaepiam maximi minimive proprietas locum habeat. Quaenam autem sit ista proprietas, ex principiis metaphysicis a priori definire non tam facile videtur: cum autem has ipsa curvas, ope Methodi directae, determinare liceat; hinc, debita adhibita attentione, id ipsum, quod in istis curvis est maximum vel minimum, concludi poterit. Spectari autem potissimum debet effectus a viribus sollicitantibus oriundus; qui cum in motu corporis genito consistat, veritati consentaneum videtur hunc ipsum motum, seu potius aggregatum omnium motuum qui in corpore projecto insunt, minimum esse debere. Quae conclusio etsi non satis confirmata videatur, tamen, si eam cum veritate jam a priori nota consentire ostendero, tantum consequetur pondus, ut omnia dubia quae circa eam suboriri queant penitus evanescant. Quin-etiam cum ejus veritas fuerit evicta, facilius erit in intimas Naturae leges atque causes finales inquirere; hocque assertum firmissimis rationibus corroborare."..... (2) "Tam late ergo hoc principium patet, ut solus motus a resistentia medii perturbatus excipiendus videatur; cujus quidem exceptionis ratio facile perspicitur, propterea quod hoc casu corpus per varias vias ad eundum locum perveniens non eandem acquirit celeritatem. Quamobrem, sublata omni resistentia in motu corporum projectorum, perpetuo haec constans proprietas locum habebit, ut summa omnium motuum elementarium sit

regards myself, I only knew in a sure manner *a posteriori* the principle I used to determine trajectories; and I have ingenuously confessed that I was not in a position to establish its truth in another manner. All that I have done is to deduce from it the same curves that are commonly found by the direct method, starting from the principles of mechanics. I have not even dared to extend its use unless I could justify by calculation its agreement with known principles. And that is what has led me to separate from this principle motions in a resisting medium and other more complicated ones; for no way presented itself to my mind of discovering the truth with regard to these motions."

Among the *Mémoires* in the Class of Speculative Philosophy in the same volume (1750) of the *Histoire*, are two by Merian[119] entitled: "Dissertation ontologique sur l'Action, la Puissance et la Liberté," and "Seconde Dissertation sur l'Action, la Puissance et la Liberté"; in the first of which[120] Maupertuis's explanation, in the *Essai de Cosmologie*, of the generation of the idea of motive force is quoted.

XIII.

In the Berlin *Histoire* for 1751, published 1753, there are five memoirs we shall have to notice, and all of the Class of Mathematics.[121]

The first is by Euler,[122] and is entitled: "Harmonie entre les Principes généraux de Repos de Mouvement de M. de Maupertuis." Both principles of Maupertuis (of 1740 and 1744) rest, says Euler, on the same foundation, so that if one is proved, the other cannot be

minima. Neque vero haec proprietas in motu unius corporis tantum cernetur, sed etiam in motu plurium corporum conjunctim; quae quomodocunque in se invicem agant, tamen semper summa omnium motuum est minima. Quod, cum hujusmodi motus difficulter ad calculum revocentur, facilius ex primis principiis intelligitur, quam ex consensu calculi secundum utramque Methodum instituti. Quoniam enim corpora, ob inertiam, omni status mutationi reluctantur; viribus sollicitantibus tamparum obtemperabunt, quam fieri potest, siquidem sint libera; ex quo efficitur, ut, in motu genito, effectus a viribus ortus minor esse debeat, quam si ullo alio modo corpus vel corpora fuissent promota. Cujus ratiocinii vis, etiamsi nondum satis perspiciatur; tamen, quia cum veritate congruit, non dubito quin, ope principiorum sanioris Metaphysicae, ad majorem evidentiam evehi queat; quod negotium aliis, qui Metaphysicam prositentur, relinquo."

[119] Pp. 459-485 and 486-516.

[120] *Ibid.*, p. 479.

[121] In this volume, the memoirs in the Classes of Experimental Philosophy and Mathematics are paged (pp. 1-356) separately from those in the Classes of Speculative Philosophy and of Belles Lettres (pp. 1-154).

[122] Pp. 169-198.

doubted. Now, Maupertuis and Euler had established the truth of the law of rest of 1740 by a multitude of different cases. Euler, then, first deduced the principle of motion from that of rest,[123] and then[124] showed that all the elementary theorems of statics follow readily from the law of rest.

The nerve of Euler's investigation is the deduction of the principle of least action from the law of rest. Euler[125] called the integral $\int V.dv$, where V is a central force acting on the body M and $v$ is the distance from M to any fixed point in the direction of V, the *effort* (effort), so that Maupertuis's law is that the sum of all the efforts is a maximum or a minimum.

"What is more natural," exclaims Euler,[126] "than to maintain that this same principle of equilibrium should also subsist in the movement of bodies under like forces? For if the intention of nature is to economize the sum of the efforts as much as possible, this intention must extend also to movements, provided that we take the efforts, not merely as they subsist in an instant, but in all the instants together for which the movement lasts. Thus, if the sum of the efforts is $\Phi$ for any instant of the motion, then, putting $dt$ for the element of the time, the integral $\int \Phi.dt$ must be a minimum. If then, for the case of equilibrium the quantity $\Phi$ must be a minimum, the same laws of nature seem to exact that, for motion $\int \Phi.dt$ should be the smallest.

"Now it is precisely in this formula that the other principle of M. de Maupertuis, concerning motion, is contained, however different it may appear at the first glance. To show this agreement, I have only to remark that when a body moves under the action of the forces V, V′, V″,...., the effort $\Phi$ to which the body is subject expresses at the same time the *vis viva* of the body— the product of the mass M of the body and the square of its velocity ($u$)." Thus the formula which must be a minimum is

$$\int M.u^2.dt = \int M.u.ds.$$

Where $v$, $v'$, $v''$,...., are the distances of M from the centers

[123] On pp. 181-182, Euler remarked that, if we wish, inversely, to deduce the principle of rest from that of motion, "we must suppose the motion infinitely small, and this causes great obscurities (*brouilleries*) in the consideration of infinitely small velocities and of the spaces which are described in an infinitely small time.

[124] *Ibid.*, pp. 183-193.

[125] *Ibid.*, p. 174.

[126] *Ibid.* p. 175.

of forces V, V′, V″,...., which are functions of these distances, Euler[127] gets the equation

$$Mu^2 = \text{const} - \Sigma \int . dv = \text{const} - \Phi;$$

and:[128] "the constant does not disturb this harmony between the effort $\Phi$ and the *vis viva* $M.u^2$ of the body; for if $\int \Phi . dt$ is a maximum or a minimum, $\int M.u^2.dt$ or $\int M.u.ds$ will be so also, since the term $\int \text{const}\, dt = \text{const}\ t$ does not enter into the consideration of the maximum or minimum. And, besides that, as the effort $\Phi$ is expressed by integral formulae, it already contains in itself any constant, so that I could have neglected this constant entirely and simply put $Mu^2 = -\Phi$, whence the identity would have been more evident. However, if we take the above integrals on a fixed footing (*sur un pied fixe*), so that the effort $\Phi$ receives a determined value, the addition of the constant will be necessary; since the velocity of the body at a certain point of its path depends on the initial velocity, and by this initial velocity the constant must be determined in each case proposed. But, of whatever quantity it may be, the determination of the maximum or minimum is not affected." Of course, as $Mu^2$ is equal to the negative of $\Phi$, if $\int Mu^2.dt$ is a minimum, $\int \Phi . dt$ will be a maximum, and reciprocally.

Euler[129] then proved "the identity between the effort and the *vis viva*" for two or more bodies, connected in any way with one another to make a flexible body: the sum of the *vires vivae* of all the elements of the body always reduces to the sum of the efforts to which all the elements are subject in the same time,—in the case of two bodies of masses M and N, distances to the[130] center of force considered $x$ and $y$ respectively, and the accelerating forces X (a function of $x$) and Y (a function of $y$) respectively,

$$\Phi = M \int X . dx + N \int Y . dy.$$

Euler[131] remarked that there are cases of equilibrium in which the sum of the efforts is a *maximum* and[132] classes the cases of equilibrium as of such natures that, if the sum of efforts is a minimum, equilibrium reestablishes itself on an infinitely small dis-

[127] *Ibid.*, p. 177.

[128] *Ibid.*, p. 178.

[129] *Ibid.*, pp. 179-181.

[130] Of course the proof extends to as many centers of force as wished.

[131] *Ibid.*, p. 194.

[132] *Ibid.*, p. 195. There is an example of the sum of efforts being a maximum on pp. 195-196.

placement being given to the system, whereas, if the sum is a maximum, this is not the case.[133]

XIV.

Euler's second paper in the volume for 1751 is entitled: "Sur le Principe de la Moindre Action."[134] This paper is concerned with the opinion that there is a minimum in the actions of nature, with Aristotle and his school, Descartes, Fermat, Leibniz,[135] Wolff, Engelhard, s'Gravesande, and others, and was occasioned by the König affair. It is ridiculous, says Euler,[136] to suppose that König's fragment was written by Leibniz, for it attributes to Leibniz a principle opposed to that which he adopted publicly in the case of the motion of light—that the product of the path described and the resistance is a minimum.

Referring to his own discovery of the minimum of the action—integral for central orbits, Euler[137] remarks: "Besides, I had not discovered this beautiful property *a priori* but (using logical terms) *a posteriori*, deducing after many trials the formula which must become a minimum in these movements; and, not daring to give it more force than in the case which I had treated, I did not believe that I had discovered a wider principle: I was content with having found this beautiful property in the movements which take place around centers of forces."

Euler's third paper in this volume is entitled: "Examen de la Dissertation de M. Le Professeur Koenig, inserée dans les Actes de Leipzig, pour le Mois de Mars, 1751."[138] In this paper Euler examined König's demonstrations with care and pronounced them to be worthless.[139]

The "Essai d'une Démonstration Métaphysique du Principe général de l'Equilibre" of Euler, printed in the same volume,[140] does not mention Maupertuis's name,[141] and is concerned with the deduction from indubitable axioms of the principle that, for equilib-

[133] Cf. Mach, *Mechanik*, pp. 70-75; *Mechanics*, pp. 69-73.
[134] *Loc. cit.*, pp. 199-218.
[135] *Ibid.*, pp. 205-209.
[136] *Ibid.*, p. 209.
[137] *Ibid.*, p. 214.
[138] *Ibid.*, pp. 219-239, "Additions," pp. 240-245.
[139] *Ibid.*, p. 220.
[140] *Ibid.*, pp. 246-254.
[141] It is, however, Maupertuis's "Law of Rest" (Cf. also Mayer, *op. cit.*, p. 23).

rium, where, P, Q,.... are forces and $x$, $y$,.... are measured on their respective lines of action,

$$\int P.dx + \int Q.dy + \ldots.$$

is a minimum.

Lastly, there is, in this volume a paper by Nicolas de Beguelin,[142] tutor of Frederick the Great's nephew who was later Frederick William II, entitled: "Recherches sur l'Existence des Corps Durs,"[143] in which Maupertuis is called a great man[144] and the illustrious author of the principle of least action,[145] and the other conclusions are just what Maupertuis would have wished.

XV.

In the Paris *Mémoires* for 1749, the Chevalier d'Arcy[146] published some reflections on the principle of least action, which he had long hesitated to publish, but that he did so in the interests of truth. D'Arcy maintained: (1) That the action of a body is not proportional to $m.v.s$, because this supposition, in a particular case, leads to a result contrary to that which the laws of motion give; (2) That, admitting Maupertuis's definition of action, the quantity of it that nature employs in each change is not a minimum, and that if in some cases this is so, the principle of least action cannot serve to prove it; (3) that Maupertuis's law of equilibrium that Maupertuis deduced from the principle of least action is only established by the introduction of a foreign and gratuitous supposition; (4) that, in general, whatever may be the laws of nature, one could always easily find a function of the masses and velocities which would represent them when it is supposed to be a minimum, but this property would not be enough to give the name of *action* to this function nor to raise the principle thence obtained to the rank of a metaphysical principle;[147] (5) that, if we define the *action of*

[142] Lived from 1714 to 1789. (Cf. Berlin *Histoire*, 1788-9 (not seen); M. Cantor, *op. cit.*, vol. iv, 1908, pp. 174 (article by F. Cajori), 227 (article by E. Netto).

[143] *Ibid.*, pp. 331-355.

[144] *Ibid.*, pp. 344, 346.

[145] *Ibid.*, p. 347.

[146] "Réflexions sur le Principe de la moindre Action de M. de Maupertuis," *Hist. de l'Acad. Roy. des Sci.*, 1749 (Paris, 1753), *Mémoires*, pp. 531-538. There is an account of this memoir in the *Histoire*, pp. 179-181. Patrick d'Arcy was born on Sept. 18 (27), 1725, at Galloway and died on Oct. 18, 1779. He was a count, a field marshal of France, and a "Pensionnaire-Géomètre" of the Paris Academy (*Poggendorffs biog.-lit. Handwörterbuch*, vol. i, p. 57). Cf. M. Cantor, *op. cit.*, vol. iv, 1908, p. 18 (article by S. Günther).

[147] *Ibid.*, pp. 535-536.

*a body around a point* to be the product $m.v.p$, where $p$ is the perpendicular drawn from this point on the direction of the *body*, then the *total action existing in nature at any instant around a given point, being produced in one given body, the quantity of action of this body will always be the same around this point*,[148] and from this theorem are easily deduced the principle of the conservation of *vis viva*, the case of rest, the centers of oscillation or of percussion, the law of the refraction of light, and so on.

With regard to (1), d'Arcy[149] gave the following considerations. "If two bodies produce equilibrium, that is to say, if rest follows from their direct impact, without our knowing to what the action is proportional, it (the action) must necessarily be equal in the two bodies; for if not, then it would follow that an action was in equilibrium with a lesser action, that is to say that different actions produce the same effect. Now, can we imagine that two equal and similar effects can be produced by unequal quantities of causes? This does not imply that the effect is proportional to its cause, but only that the same effect is always produced by the same quantity of cause and *vice versa*.

"Let there be two hard bodies A and B perfectly equal and proceeding in opposite directions with equal velocities, then clearly rest will follow their impact. If A, proceeding in the same direction with the same velocity, is impinged upon by the body C of different mass and velocity, but such that rest follows impact, I believe that nobody can deny that the action of B is equal to that of C, since both destroy the velocity of A. Can we have another idea of the equality of two quantities than of our being able to substitute one for the other without changing anything?" If B proceeds with double the velocity of, and traverses double the space traversed by, C, the principle of Maupertuis says that the mass of C is four times that of B; and this is contrary to what we find by the laws of motion. "Thus," concludes d'Arcy, "the action is not proportional to the mass multiplied by the velocity and by the space described."

With respect to (2), d'Arcy[150] remarked that if two bodies A and B proceed in the same direction with the velocities $a$ and $b$,

[148] This theorem was given by d'Arcy in the Paris *Mémoires* for 1747 (published in 1752; pp. 348-356) under the title: "Principe général de Dynamique, qui donne la relation entre les espaces parcourus et les temps, quelque soit le système de corps que l'on considère, et quelles que soient leurs actions les uns sur les autres." This memoir (read in 1746) is part of the paper (of three memoirs) entitled: "Problème de Dynamique" on pp. 344-361.

[149] *Loc. cit.*, pp. 532-533.

[150] *Ibid.*, pp. 533-534.

the action of the bodies A and B will be $Aa^2+Bb^2$. If after impact they proceed with the velocities $x$ and $z$, their action after impact will be $Ax^2+Bz^2$.[151] Now the quantity of action after impact will be either equal to or less than or greater than what it was before impact: if it is equal we have the theorem of *vis viva*, which does not hold for hard bodies; if it is greater it will have increased by the quantity

$$Ax^2+Bz^2-Bb^2-Aa^2;$$

if it is smaller it will be diminished by the quantity

$$Aa^2+Bb^2-Ax^2-Bz^2,$$

and this quantity is the real quantity of action lost, and consequently is that employed by nature to produce the actual change; therefore

$$2Ax.dx+2Bz.dz=0,$$

or, if we suppose $dx=dz$,[152]

$$Ax+Bz=0,$$

which is absurd. It is not, then, the destroyed part of this quantity which is a minimum. Maupertuis's argument is: Suppose that the bodies A and B proceed in the same direction with the velocities $a$ and $b$ and that the plane on which they are moves with the velocity $x$; evidently A will move on this plane with a velocity $a-x$ and B will move behind with a velocity $x-b$, $x$ being greater than $b$ and less than $a$. Maupertuis finds that

$$A(a-x)^2+B(x-b)^2$$

will be a minimum when the velocity $x$ is such that

$$A(a-x)=B(x-b),$$

that is to say, when the bodies are in equilibrium on this plane. "I vow," said d'Arcy,[153] "that I do not know what consequence one can deduce from this other than: $AP^2+BQ^2$ being a minimum and $P^2=\int\Phi.dx$ and $Q^2=\int\Delta.dx$, we will have

$$A.\Phi+B.\Delta=0,$$

and consequently if

$$A.Z=B.X,$$

where Z and X are functions of $x$, then $AZ^2+BX^2$ will always be a minimum, and *vice versa*; and this leads me to believe that, when one has found that $A.Z^2+B.X^2$ is a minimum, one knew that $A.Z=B.X$."

[151] "Since $a$, $b$, $x$ and $z$ express the spaces as well as the velocities."

[152] For hard bodies $x=z$ and for elastic ones $a-b=z-x$.

[153] *Ibid.*, p. 534.

With regard to (3), when Maupertuis deduced the law of the lever from his principle of least action, he made a gratuitous supposition that the lever moves with a constant angular velocity.[154] To find the point of the lever (of length C) about which two bodies of masses A and B at the ends of the lever produce equilibrium, Maupertuis called Z the distance of A to the sought point, and announced that, to solve the problem, he would suppose the lever to receive some small movement and then express that the quantity of action is the smallest possible. If, remarked d'Arcy, we call V the small velocity of A and suppose that A describes a space $a$, the velocity of B and the space described by it will be, respectively,

$$V(C-Z)/Z \text{ and } a(C-Z)/Z,$$

and the action of the bodies will be

$$AVa + BVa(C-Z)^2/Z,$$

and the differential equated to zero, supposing that $a$ and V are constant, gives $Z=C$. Maupertuis gets the correct law by supposing that the lever moves with a constant angular velocity. But this supposition, says d'Arcy, "seems to me absolutely gratuitous, since, to each value of Z, the action or the time necessary for it to describe the constant angle is different."

With regard to (5), d'Arcy[155] remarks that his definition of action is in perfect agreement with d'Alembert's:[156] "The action is the movement that a body produces or tends to produce in another body."

D'Arcy's principle is that the sum of the masses of a system, each mass being multiplied by the sector which it describes around a fixed point in the same time, less the sum of the sectors described in the contrary sense, each being multiplied by the mass of the body which describes it, is proportional to the time. The only difference from the principle that d'Arcy gave in this memoir of 1749 is that instead of (as in 1747) sectors multiplied by masses, were used in 1749 the equivalent expressions $m.v.p$.

Let two bodies A and B move with the velocities $a$ and $b$ before impact and with the velocities $x$ and $z$ after impact. By the above principle the action of A and B round any point O will be the same after as before the impact; thus, where P is the foot of the perpendicular from O on the line joining A and B,

[154] *Ibid.*, p. 535.
[155] *Ibid.*, p. 536.
[156] In the *Encyclopédie* (not seen).

$$A.a.OP+B.b.OP=A.x.OP+B.z.OP,$$

and consequently

$$A(a-x)=B(z-b),$$

and this relation between the velocity lost by A and that gained by B holds whether the bodies are elastic or not. In elastic bodies we easily see that $a-b=z-x$, and hence, from the above equation

$$A(a^2-x^2)=B(z^2-b^2),$$

which is the property of *vires vivae.*[157]

If two bodies A and B strike the ends P and Q of a straight lever with the same velocity $a$, to find the fulcrum-point C of the bar such that A and B remain at rest after the impact, d'Arcy[158] observes that the action of A round C must be equal to the action of B round C, and thus C is the Center of gravity. By the same method we find the centers of oscillation or of percussion, and so on.

When deducing the law of the refraction of light,[159] d'Arcy observes that, in his memoir of 1747, he had proved that it is the same thing whether the bodies are attracted toward the point round which the action is sought or not, as the quantity of this action is not altered thereby. Let FG be the surface of a diaphanous and homogeneous sphere of center C, M a point outside the sphere, and N a point inside. A ray of light—$\mu$ being the mass of a corpuscle of light—travels from M to N, its velocity outside the sphere being $v$ and inside the sphere being $u$, meeting the surface at $m$. "The action of the surface FG can only be towards the center C; for whatever action this body may have on the corpuscle of light on one side of the perpendicular to the surface, it will have the same action on the other side." Thus we have

$$\mu.v.CR=\mu.u.Cr,$$

and this gives the known law of refraction of light. The case of FG being plane instead of spherical is then treated, and d'Arcy finally remarks that other examples of the application of his principles are given in the memoir of 1747.

XVI.

The Berlin *Histoire* for 1752, published in 1754, contains among the memoirs of the class of Speculative Philosophy a "Réponse à un Mémoire de M. d'Arcy inséré dans le Volume de l'Académie Royale

[157] D'Arcy, *loc. cit.*, p. 537.
[158] *Ibid.*
[159] *Ibid.*, pp. 537-538.

des Sciences de Paris pour l'année 1749" by Maupertuis,[160] which is headed by a notice,[161] in italics, stating that the memoir (1744) in which the principle of the least quantity of action was first communicated was received by the Paris Academy, Maupertuis "dares to say, with some applause (*applaudissment*)." Then Maupertuis refers to his paper of 1746, to his *Essai de Cosmologie,* to the attacks of "un Professeur de la Haye" to whom, as he used libels, he will never reply, and to d'Arcy who "attacks with so much politeness and modesty," that Maupertuis thinks that he ought to reply to him. He appears, says Maupertuis, "to be such a lover of the truth that I will try to introduce him to it."[162]

(1) D'Arcy tried to show that Maupertuis is wrong to call *m.v.s action.* Maupertuis believed that he had good grounds for justifying the name; but, to cut matters short, Maupertuis said that he had adopted Leibniz's definition.[163] D'Arcy's reason against calling the above product *action* reduces to this: In the impact of hard bodies, two different quantities of *action* reduce to rest one and the same body moving with the same velocity. By the same kind of reasoning, says Maupertuis, d'Arcy might object to the name *vis viva*; for two different *vires vivae* can reduce the same hard body to rest." And in fact here the *vis viva* is the same as the *action,* for here "the space is proportional to the velocity."[164] Again, with elastic bodies, if two unequal bodies with the same *vires mortuae* (*m.v*) strike a third body at rest, different *vires mortuae* will come into existence or perish.

(2) D'Arcy, to show that Maupertuis is wrong in holding that the quantity of action necessary to produce any change in nature is a minimum, confuses, when treating of impact, change of the quantity of action with change of velocities.[165] The velocities can change without the quantity of action changing, as is the case in the impact of elastic bodies (when this quantity is the same as the quan-

[160] *Histoire de l'Acad.* de Berlin, 1752, T. VIII, pp. 293-298.

[161] *Ibid.,* pp. 293-294.

[162] "....et paroit si Amateur de la vérité, que je tâcherai de la lui faire connoître" (*ibid.,* p. 294).

[163] "....mais pour trancher court avec M. d'Arcy, je puis dire que ce n'est pas mon affaire. Leibnitz, et ceux qui l'ont suivi, ont appelé ainsi le produit du corps par l'espace et par la vitesse; j'ai adopté une définition établie, contre laquelle on n'avoit point disputé, et que je n'avois aucune raison de changer; voilà ce qu'il me suffiroit de répondre"; *ibid.,* p. 295. Apparently this is upon what E. du Bois-Reymond relies when he says (*op. cit.,* p. 48): "Maupertuis borrowed, as he himself says, the concept and name of *action* from Leibniz..."

[164] *Ibid.,* p. 295.

[165] *Ibid.,* p. 296.

tity of *vis viva*); in the impact of hard bodies, the change of the velocities is neither equal nor proportional to the change in the quantity of action.

If[166] the bodies are elastic, the change is: A which moved before with the velocity $a$ moves afterwards with the velocity $\alpha$, and the corresponding velocities of B are $b$ and $\beta$. If then we wish that afterwards A should move with the velocity $a$ and B with the velocity $b$, we must transport the A-plane with the velocity $a-\alpha$ and the B-plane with the velocity $\beta-b$; and from this we must get the quantity of action $A(a-\alpha)^2+B(\beta-b)^2$ necessary to produce the change in nature, and which is a minimum. If A and B are hard, and the common velocity after the impact is $x$, and if we wish each body to move with its original velocity, we proceed as before, and get, for the quantity of action necessary to produce this change, $A(a-x)^2+B(x-b)^2$, the smallest possible.

(3) D'Arcy's criticism on Maupertuis's deduction of the lever is mistaken, for Maupertuis supposed the lever to be in a state of rest and infinitely little displaced from this state.[167]

Finally, Maupertuis[168] mentioned the incompleteness of this theory of the lever, which was not remarked by d'Arcy, but about which we have read in connection with the reprint of the memoir of 1740[169] in Maupertuis's *Œuvres*.[170]

XVII.

In the Paris *Mémoires* for 1752 appeared a reply by d'Arcy[171] to Maupertuis's paper in the Berlin *Mémoires* for 1752. After a few preliminary words in which what looks like sarcasm is veiled in words of compliment—Maupertuis's "modesty," "politeness," and "simplicity" being praised, d'Arcy[172] confesses that if he had need of a proof of an arranging intelligence he would find it just as much in the uniformity of the laws of generation of the vilest insects as in the general laws of mechanics.

[166] *Ibid.*, pp. 296-297.

[167] *Ibid.*, pp. 297-298.

[168] *Ibid.*, p. 298.

[169] Mapertuis here refers to this paper as being in the *Mémoires* for 1743. This is, of course, a misprint.

[170] See section II above.

[171] "Replique à un Mémoire de M. de Maupertuis, sur le principe de la moindre action, inséré dans les Mémoires de l'Académie royale des Sciences de Berlin, de l'année 1752," *Hist. de l'Acad. Roy. des Sci.*, 1752 (Paris, 1756), *Mémoires*, pp. 503-519.

[172] *Ibid.*, p. 503.

With regard to Maupertuis's (correct) classification of d'Arcy's objections under three heads, d'Arcy[173] maintains that the first still holds, for "when someone says that nature economizes action, he clearly means that this quantity expresses this cause or the real force," and d'Arcy[174] even accuses Maupertuis of falling back on the authority of Leibniz. His argument depends for its validity on the principle that a definition is something more than the mere giving of a name.

With regard to d'Arcy's second objection, d'Arcy[175] quoted from the *Encyclopédie*[176] to show that Maupertuis's phrase "change happened in nature" and that his own interpretation of this phrase in the above simple case of impact as

$$Aa^2 + Bb^2 - Aa^2 - B\beta^2,$$

which is to be a minimum, so that

$$Aa + B\beta = 0,$$

is natural and also showed[177] that Maupertuis himself implied this interpretation.

Then d'Arcy[178] showed that the manner in which Maupertuis used his principle in the case of the refraction of light is different from that in which he used it in the case of impact. If we treated the latter case like the former, we should have the result that

$$Aa^2 + Bb^2 + Aa^2 + B\beta^2$$

is a minimum, and hence that

$$Aa^2 + B\beta^2 = 0.$$

In the case of light, it is the action before the change plus the action after the change which is a minimum; in impact it is the mass by the velocity lost and by the space which will be described in consequence of this velocity.

With respect to Maupertuis's reply to d'Arcy's third objection, Maupertuis, says d'Arcy,[179] has misread the objection: there was not said to be a supposition about an *angular and constant* motion but about a *constant angular* motion. D'Arcy quotes objections nearly the same as his of 1749 from the above cited article on "Cosmo-

[173] *Ibid.*, p. 504.
[174] *Ibid.*, p. 506.
[175] *Ibid.*, pp. 507-508.
[176] Article "Cosmologie," p. 196 [not seen].
[177] D'Arcy, *loc. cit.*, pp. 508-509.
[178] *Loc. cit.*, pp. 509-510.
[179] *Ibid.*, pp. 510-511.

logie": "When Maupertuis applies his principle to the case of equilibrium in the lever, certain suppositions must be made, amongst others, that the velocity is proportional to the distance from the fulcrum,[180] and that the time is constant as in the case of impact...."

In the case of the reflection of light, d'Arcy[181] shows that nature is prodigal or avaricious of action as a mirror is more or less concave respectively, and again quoted the article "Cosmologie" on this point.

Finally, d'Arcy[182] returned to his principle of 1747, which he prepared to substitute for Maupertuis's principle.[183]

XVIII.

In the Berlin *Histoire* for 1753, published in 1755, the only paper[184] relating to the principle of least action is an "Examen des Reflexions de M. le Chevalier d'Arcy sur le Principe de la moindre action" by Louis Bertrand.[185] Bertrand's paper was headed by a note to the effect that, as the Paris Academy of Sciences had, contrary to its custom, hurried to publish in its *Mémoires* of 1749 some reflexions of d'Arcy which he had only given in 1752, the Berlin Academy believed that it might publish this examination one year before it ought to have appeared.

D'Arcy, says Bertrand,[186] undertook to overthrow Maupertuis's principle, but only succeeded in overthrowing the false ideas which he had formed about it. In the first place, d'Arcy objected to Mau-

[180] As d'Arcy expressed it, that the angular velocity is constant.

[181] *Ibid.*, pp. 511-513.

[182] *Ibid.*, pp. 513-519. On p. 513 he emphasized that the memoir containing this principle was read to the French Academy in 1746.

[183] On d'Arcy's memoirs see Mayer, *op. cit.*, pp. 13-15, 21. It seems to me that Mayer's view of these memoirs is too favorable. I will return to this point in my criticisms.

[184] The contrary was stated, owing to a wrong reading of A. Mayer, *op. cit.*, p. 17, by myself in *Ostwalds Klassiker*, No. 167, p. 36; but, of Euler's five papers in this volume, one is on Daniel Bernoulli's papers on vibrating cords (cf. M. Cantor, *op. cit.*, vol. iii, 2d ed., Leipsic, 1901, pp. 904-907), two papers are on spherical and spheroidical trigonometry deduced from the method of maxima and minima (cf. *ibid.*, pp. 867-869), one on the law of refraction of rays of different colors, and one on the paths of projectiles in resisting media;—and in none of these is any reference to the principle of least action except in a passage (p. 306) in the last line but one of these papers, where he refers to the convincing proof of the existence of a Deity given by Maupertuis, and also to the argument from the wonderful structure of the eye.

[185] Pp. 310-320. Louis Bertrand (1731-1812) was then in Berlin and was a friend of Euler's; cf. *Poggendorff*, vol. i, p. 171; M. Cantor, *op. cit.*, vol. iv, Leipsic, 1908, p. 332 (article by V. Bobynin).

[186] *Op. cit.*, p. 311.

pertuis's definition of action. This is a question of words;[187] d'Arcy required that the *action* of different hard bodies should be estimated equal if each of these bodies is capable of reducing to rest the same hard body endowed with a certain velocity, so that the *action* of a body is measured by $m.v$. But, says Bertrand, it is well known[188] that, in the impact of hard bodies, a part of the motion is destroyed —that part which would be reproduced if the bodies were elastic: "hence it follows that, if a hard body (A) of mass 1 and velocity 1 were reduced to rest both by a body (B) of mass 1 and velocity 1 and by a body (C) of mass ½ and velocity 2, we could only affirm positively that the action of B is equal to that of C if we have previously proved that when B impinges on A it loses the same quantity of motion as when C impinges on A. For if it were true that in one case more motion were lost than in the other, the rest in this case ought not to be attributed to the equality of action of the two bodies, but to the greater loss of motion; in fact, if this loss had not been greater, some motion would have been left for the bodies which have impinged, and thus rest would not have followed the impact.

"In order, then, that the reasoning by which M. d'Arcy has wished to prop up his definition of *action* should be conclusive, it would be necessary for him to prove that the same quantity of motion is lost whether B impinges on A or C impinges on A. Now this he will never prove.

"Not being able to do anything in that direction, perhaps he will claim that it is sufficient to attend to the change which happens to the body A after the impact; but, if he only pays regard to the effect produced on the body impinged upon, we can urge against him the impact of elastic bodies, where a body A of mass and velocity both 1 is reduced to rest both by a body B of mass 1 and velocity 0, by a body C whose velocity and mass are both ½, and by a body D whose mass is ⅓ and velocity 1. Now, M. d'Arcy would contradict his own definition of action if he claimed that the actions of B, C, and D were all equal to one another. Thus the foundation on which M. d'Arcy wished to support his manner of estimating action absolutely lacks solidity." In d'Arcy's last paragraph on the definition of action, he wrongly concludes, says Bertrand,[189], that from Maupertuis's definition of *action,* follows that whenever the

[187] *Ibid.,* pp. 311-312, 313.

[188] "....C'est une chose dont tous les Philosophes conviennent...." (*ibid.,* p. 312).

[189] *Ibid.,* pp. 313-314.

velocities and the masses of two hard bodies are such that rest follows the impact of these bodies, the actions of these bodies are equal.

With regard to d'Arcy's attack on Maupertuis's principle, Bertrand[190] remarks that Maupertuis expressly said that not the difference of the actions before and after the impact, but the quantity of action necessary to produce this change is to be a minimum. The quantity of action necessary to produce a change is not the difference of the actions before and after the change; but it is the product of the mass of the bodies whose state is changed, the space that these bodies describe in consequence of (*en suite du*) the change, and the velocity with which they describe it, also in consequence of the change.[191]

With regard to d'Arcy's strictures on Maupertuis's treatment of the lever, Bertrand[192] reproduces d'Arcy's supposition that A moves with a small velocity V and describes a space $a$, whence the velocity of B is $V(c-z)/z$, the space described by B is $a(c-z)/z$, and the action of the whole system is

$$AVa + BVa(c-z)^2/z^2.$$

Then, before differentiating, d'Arcy supposed V and $a$ constant; and Bertrand inquires why should the velocity of and space described by A be supposed to be constant rather than those of and by B. Maupertuis puts as constant the angle that A and B describe around the fulcrum of the lever; and this supposition does not affect one of the bodies rather than the other, for this angle is the same for both bodies. Still, this supposition appears gratuitous to d'Arcy because for each value of $z$ the action or the time necessary to make A and B describe the angle supposed constant is different. But, says Bertrand, if the action necessary to make A and B describe the angle supposed constant were not different for each value of $z$, it would be absurd to seek which of these actions is the least.

With regard to d'Arcy's assertion that, whatever the laws of nature might be, it would always be easy to find a function of the velocities and masses such that, when minimized, it would give these laws, Bertrand[193] remarks that "that may be true of many particular cases." Rather earlier in his paper, Bertrand[194] remarks *à propos*

[190] *Ibid.*, p. 314.

[191] *Ibid.*, pp. 314-315.

[192] *Ibid.*, pp. 317-318.

[193] *Ibid.* p. 318.

[194] *Ibid.*, pp. 315-316.

of d'Arcy's suggestion that Maupertuis knew the formula $A(a-x) = B(x-b)$ for impact and concluded that the action must be

$$A(a-x)^2+B(x-b)^2$$

in order that the known formula should result when the action was minimized, and d'Arcy's attempted generalization, that, if Z and X are functions of $x$, then, if $AZ=BX$,

$$AZ^2+BX^2$$

will always be a minimum and *vice versa*, that this generalization will always be false except when $dZ+dX=0$,—the case which he wished to generalize.

The rest of Bertrand's[195] paper is devoted to d'Arcy's own principle. "This principle," says Bertrand,[196] "can in a certain sense be admitted, but it will never lead to important discoveries; still less will it show us, so to speak, the true ends in view of nature: and these circumstances put it infinitely below that of M. de Maupertuis."

With regard to the way in which Bertrand's paper is written, it seems both magisterial and hasty: attempts at sarcasm against d'Arcy and flattery—or perhaps sincere reverence—for Maupertuis stand out too prominently. Bertrand was young when he wrote it, so there is a greater chance that he was sincere. Still, he was of, or was about to be of, the Berlin Academy.

## XIX.

We will now give a brief retrospect of the development of views on the principle of least action, and dispose of all historical questions before trying to elicit what gains have resulted for knowledge by this development.

A. Mayer[197] says of Euler's formulations of the principle of least action: "We shall see that this correct form [in the second appendix to the *Methodus* of 1744] got lost to him in the course of time, and that soon it lost as much in rigor as it appeared to gain in generality." Mayer's[198] grounds for this view were that Jacobi's[199] principle of least action was the "true" principle, owing to

[195] *Ibid.*, pp. 318-320. Just at the end is: "On pourroit faire encore nombre de réfléxions sur l'insuffisance de ce Principe appliqué à la réfraction des rayons de lumiére; mais il semble qu'il y auroit une sorte de mauvaise humeur à examiner si rigoureusement se que M. d'Arcy paroit avoir voulu traiter cavaliérement." I have left the accents unaltered.

[196] *Ibid.*, p. 319.

[197] *Op. cit.*, p. 6.

[198] *Op. cit.*, pp. 6-11.

[199] Cf. *Monist*, vol. xxii, April, 1912.

the difficulty there appeared to be[200] if the time was not eliminated, and this elimination had apparently to be done by the equation expressing the conservation of *vis viva*. Thus the principle of least action is subject to the limitations implied by the subsistence of the theorem of *vis viva*. Thus Euler, in the above mentioned appendix, expressly pointed out that his theorem cannot hold for motion in a resisting medium, and that, in the integrand, the velocity must be expressed "ex viribus sollicitantibus per quantitates ad curvam pertinentes.[201] Consequently Mayer[202] maintained that Lagrange's (1760) generalization of the principle of least action is, in the form in which Lagrange states it, meaningless, and the theorem which he really had in his mind is that known as "Hamilton's principle" given by Hamilton in 1835. We know[203] that later on (in a publication of 1886) Mayer changed this view, owing to acquaintance with a paper of Rodrigues's (1816) in which the time (the $t$ in the integrand) was varied by the $\delta$-process of the calculus of variations, and admitted that there are two forms of the principle of least action: Jacobi's and Lagrange's. This view has been confirmed by the later researches of Hölder.[204]

Now Jacobi's principle may be considered to be a generalized form of Euler's theorem, and Lagrange's principle a more precise and generalized form of Maupertuis's. So it happens that Maupertuis was right in thinking his theorem quite general, and Euler

[200] *Ibid.*

[201] *Methodus*, p. 312. Cf. pp. 318-319 on the necessity for the principle of *vis viva*.

[202] *Op. cit.*, pp. 26-29. Mayer (*ibid.*, p. 24) also remarked that Euler's later (Maupertuisian) form of the principle, in which the condition that all the quantities in the integrand must be reduced, by means of the principle of *vis viva*, to space-elements alone is not stated, is quite meaningless, for the forces acting on the system, on which the path of the system depends, do not occur in the integral of action. Here we will anticipate our criticism by pointing out that in Lagrange's memoir the conditoin

$$\delta T = \delta U,$$

where "T" and "$\delta$U" have the meaning already explained in *The Monist*, vol. xxii, April, 1912, p. 290, is *explicitly* given, and what would now be written in the same way was, tacitly or not, presupposed in all Euler's works. Mayer said that the problem of variations only subsisted under the condition

$$T = U + \text{const.},$$

which implies the preceding equation, but, as Lagrange pointed out, is not necessarily implied by it. And it is the preceding equation alone that we require to rescue the principle of least action from meaninglessness. Mayer's remark (*ibid.*, p. 27) that Lagrange completely leaves out the condition is simply an error.

[203] Cf. *Monist*, vol. xxii, April, 1912.

[204] *Ibid.*

was right in doing what Mayer[205] complains of—in dropping the condition about the theorem of *vis viva* holding.[206] Of course, it may have been, and probably was, the case that neither Maupertuis nor Euler had any good grounds for believing that they were right. Indeed, one is forced, against one's will, to the opinion that Euler was in a position in which, as Mayer[207] expresses it, "he could not with propriety retort to the powerful President of his Academy."

The only reason why it is necessary to inquire closely whether Euler really considered Maupertuis's principle to be valid seems to me mainly to be the provision of an example to show the necessity of an additional condition when we wish to deduce properties of motion from the equation of the variation of the integral of action to zero. There is also the possibility of our being given yet another example of the greater power of instinctive beliefs or "metaphysics" over the good man's mind than the love of scientific truth.[208] If we should have to conclude that Euler deliberately hid the truth for the personal favor of Maupertuis, this conclusion will fill us with the same regret and loathing that we feel for the weakness in Galileo's character and the disgraceful exercise of the church's power on him, respectively.

It seems to me true that Euler's love for "metaphysics" alone could not lead him to forsake scrupulous honesty in his search for the truth. It is difficult, but very possible, to acquit Euler of the charge of veiled sarcasm against Maupertuis's principle. In a paper, from which we have quoted above, in the Berlin *Mémoires* for 1748, he expresses his belief that we are still very far from being able to assign, for each effect which nature produces, the quantity of action which is the smallest, and from being able to deduce it from the first principles of our knowledge. Indeed, Euler seems to have no doubt that *something* must be a minimum, but he also thinks that this something may be different— or at least seem to us, without imperfect knowledge, different—in different cases.[209] At any rate Euler goes carefully through single statical cases and determines the equivalent in terms of the forces of "the quantity of

[205] *Op. cit.*, pp. 23-24. Euler did not, however, *explicitly* drop this condition.

[206] Euler had presupposed in 1744 that the principle of *vis viva* held: Maupertuis considered his principle applies to cases—such as the impact of inelastic bodies—where the principle of *vis viva* does not hold.

[207] *Ibid.*, p. 17.

[208] On Euler's "metaphysical" tendencies, cf. Mayer, *ibid.*, pp. 21-23.

[209] Cf. the remark of d'Arcy that, whatever the laws of nature might be one could always find a function of the masses and velocities which, when minimized, would represent them (cf. section XV).

action" in each case. Nowadays, we would say,[210] of course, that this inductive procedure was far more "reasonable" or scientific than Maupertuis's; but we must remember that then the opinion was far more generally held than it is now that knowledge of the truth could be attained by other than scientific methods.

It was, I think we must say, not merely love for "metaphysics" which led Euler to sacrifice important details of his principle. Comparison of Daniel Bernoulli's letter to Euler of September 4, 1743, with Euler's markedly different account in the Berlin *Mémoires* of 1750 of the circumstances about the publication of the *Methodus* of 1744, as well as Euler's obviously unjust attitude towards König, points to a lower influence. If we dismissed—as we would like—thoughts that this sort of influence came in, we would be faced with the insoluble problem that Euler supported a principle which was claimed to embrace cases where the theorem of *vis viva* fails while he had convinced himself that the subsistence of this theorem was a necessary condition for the validity of the principle. And here the suggestion arises of itself that, since Euler, in his papers in the Berlin *Mémoires*, only committed himself to the mathematical support—as distinguished from warmly expressed admiration—of Maupertuis's principle in *statical* cases, he dared not affirm that the action-integral was a minimum in nature even when the principle of *vis viva* did not hold.[211] This stop was reserved for Lagrange, and perhaps it was on this account that Euler in a letter of November 9, 1762, congratulated Lagrange in the words:[212] "What satisfaction would M. de Maupertuis not have, if he were still alive, to see his Principle of least action carried to the highest degree of dignity of which it is susceptible.[213] If this conjecture be true, we must believe that Euler had a childlike faith that "metaphysics" could generalize a theorem so far as to drop a condition which he had satisfied himself, was necessary. We know now that this faith —if indeed it existed—was justified.

PHILIP E. B. JOURDAIN.

THE LODGE, GIRTON, CAMBRIDGE, ENGLAND.

[210] Like Mayer, *op. cit.*, p. 23.

[211] Indeed, where he refers to dynamical cases (in the Berlin *Histoire* of 1751) he explicitly uses the principle of *vis viva*. Euler nowhere refers to the problem of the impact of inelastic bodies, on which Maupertuis and others laid such stress.

[212] *Œuvres de Lagrange*, vol. xiv, p. 201.

[213] "Quelle satisfaction n'aurait pas M. de Maupertuis, s'il était encore en vie, de voir son principe de la moindre action porté au plus haut degré de dignité dont il est susceptible."

## THE CAPTURE HYPOTHESIS OF T. J. J. SEE.[1]

In the opinion of Mr. See,[2] the planets were not formed from fragments of the solar nebula, nor did the moon originate from a piece of that of the earth. He believes that the planets had a cosmic origin outside of the solar nebula; that they are foreign bodies captured by the sun while passing near it in their journey; and that in the same way the moon was captured by the earth at a certain remote time.

How was this phenomenon accomplished? Mr. See thinks that the sun was formerly surrounded by a vast atmosphere and that the capture took place as the result of a resistance created by this atmosphere.

Let us therefore study the effect of the resistance of the medium on the motion of a planet.[3] If there were no resistance the motion would be Keplerian, the orbit would be an ellipse of any eccentricity whatever. The density of the resisting medium being by hypothesis very small, this orbit would vary slowly. We shall study the variations of this orbit by the method of the variation of constants.

First let us recall some formulas pertaining to the elliptical motion of planets.

Calling the radius vector $r$ and the true anomaly $v$, the equation of the orbit is

$$r=\frac{p}{1+e\cos v}, \tag{1}$$

$e$ denoting the eccentricity, and

$$p=a(1-e^2) \tag{2}$$

denoting the parameter of the elliptical orbit whose major axis is $2a$. We have also the equation of the areas

$$r^2\frac{dv}{dt}=C,$$

the constant C of the areas having the value

$$C=\sqrt{Mp},$$

in which M represents the mass of the sun. (We disregard the

[1] Translated by Lydia G. Robinson from the author's *Leçons sur les hypothèses cosmogoniques*, Chaps. VI and XIII. Paris, Hermann, 1911.

[2] T. J. J. See, *Researches on the Evolution of the Stellar Systems*, Vol. II, "The Capture Theory of Cosmical Evolution." Lynn, Mass., Nichols & Sons; Paris, Hermann, 1910.

[3] See *loc. cit.*, Chap. VII, pp. 134-158.

mass of the planet compared to the sun's mass.) The mean motion $n$ is connected with half the major axis $a$ by Kepler's third law.

(3) $$n^2a^3=\mathrm{M}.$$

Finally the equation of the *vis viva* gives

$$\mathrm{T}-\frac{\mathrm{M}}{r}=-\frac{\mathrm{M}}{2a},$$

in which T is half the *vis viva*.

Differentiating equation (1) with reference to time, we have

$$\begin{aligned}\frac{dr}{dt}&=\frac{pe\sin v}{(1+e\cos v)^2}\frac{dv}{dt}\\&=\frac{pe\sin v}{(1+e\cos v)^2}\frac{\mathrm{C}}{r^2}\\&=\frac{pe\sin v}{(1+e\cos v)^2}\frac{\mathrm{C}}{p^2}(1+e\cos v)^2\\&=\frac{\mathrm{C}}{p}e\sin v.\end{aligned}$$

Now $dr/dt$ is the component of velocity in the direction of the radius vector. The component perpendicular to this radius vector has for its value

$$\begin{aligned}r\frac{dv}{dt}&=\frac{\mathrm{C}}{r}\\&=\frac{\mathrm{C}}{p}(1+e\cos v).\end{aligned}$$

From the two components of the velocity V, we derive the square of this velocity,

$$\mathrm{V}^2=\frac{\mathrm{C}^2}{p^2}(1+2\,e\cos v+e^2).$$

In short, if we put

$$\rho^2=1+2\,e\cos v+e^2,$$

we shall have

$$\begin{aligned}\mathrm{V}&=\frac{\mathrm{C}}{p}\rho\\&=\rho\sqrt{\frac{\mathrm{M}}{p}}.\end{aligned}$$

The above formulas belong to Keplerian motion.

Now let us suppose that there is an atmospheric medium with a resistance R directly opposed to the velocity and function of the

value V of that velocity. The constant of the *vires vivae* – M/2*a* during the time *dt* will undergo a variation

$$\frac{M}{2a^2}da;$$

this variation will equal the work of the resistance R which is

$$-RV dt.$$

Hence we have

$$\frac{M}{2a^2}\frac{da}{dt} = -RV$$

$$= -R\rho\sqrt{\frac{M}{p}},$$

whence we derive

$$\frac{da}{dt} = -\frac{2R\rho a^2}{\sqrt{Mp}};$$

replacing M and *p* by their values (2) and (3) in this last equation, we obtain

$$\frac{da}{dt} = -\frac{2R\rho}{n\sqrt{1-e^2}}. \tag{4}$$

This is the equation which gives the variation of the major axis; the second member is necessarily negative. Hence the effect of the resistance of the medium is always to diminish *a* and consequently according to equation (3) to increase *n*. The angular velocity of the planet increases[4] at the same time that its mean distance from the sun diminishes.

W shall now study the effect of resistance of the medium on the eccentricity of the orbit.

First of all the derivative *dC/dt* of the areal constant C would be equal to the momentum of the disturbing force R, with reference to the center of attraction. Now this force R opposed to the velocity has for its components:
in the direction of the vector ray

$$-R\frac{\frac{dr}{dt}}{V},$$

perpendicular to the vector ray

[4] Formula (3) even shows that *na* increases as *a* diminishes, whence we have the curious result that resistance of the medium causes an increase in the linear velocity of the planet.

$$-R\frac{r\frac{dv}{dt}}{V};$$

and the momentum of the force R with reference to the sun is

$$-R\frac{r^2\frac{dv}{dt}}{V}=-R\frac{C}{V}.$$

Hence we have

$$\frac{dC}{dt}=-\frac{RC}{V}. \tag{5}$$

Remember that

$$C=\sqrt{Mp}$$
$$=M^{\frac{1}{2}}a^{\frac{1}{2}}(1-e^2)^{\frac{1}{2}}.$$

Taking the logarithmic derivatives of the two extreme members, we have

$$\frac{dC}{C}=\frac{1}{2}\left(\frac{da}{a}-\frac{2e\,de}{1-e^2}\right).$$

This equation makes it possible for us to obtain $de$ since $da$ and $dC$ have been computed. We find

$$\frac{2e}{1-e^2}\frac{de}{dt}=\frac{1}{a}\frac{da}{dt}-\frac{2}{C}\frac{dC}{dt},$$

an equation which may be written by replacing $da/dt$ and $dC/dt$ by their values (4) and (5),

$$\frac{2e}{1-e^2}\frac{de}{dt}=-\frac{2R\rho}{na\sqrt{1-e^2}}+\frac{2R}{V}. \tag{6}$$

Let us now transform the second member of this equation. We have previously found (page 461)

$$V=\rho\sqrt{\frac{M}{p}}$$
$$=\rho\frac{na}{\sqrt{1-e^2}};$$

hence the second member may assume the form

$$-\frac{2R}{na\sqrt{1-e^2}}\left[\rho-\frac{1-e^2}{\rho}\right],$$

or again, by restoring the value of $\rho^2$, this other form

$$-\frac{2R}{na\sqrt{1-e^2}}\frac{2e\cos v+2e^2}{\rho}.$$

Finally equation (6) then gives

$$\frac{de}{dt}=-\frac{2R\sqrt{1-e^2}}{nap}(e+\cos v). \tag{7}$$

This is the equation which gives the variation of the eccentricity of the orbit.

Formulas (4) and (7) make it possible to compute at any instant the variations of the major axis and of the eccentricity. But here it is only desirable to obtain their *secular* variations, and in order to do this, to compute the value of $da$ and $de$ during the time of a complete revolution.

Taking as an independent variable the true anomaly $v$ we shall have

$$\begin{cases}\frac{da}{dv}=\frac{da}{dt}\frac{dt}{dv},\\ \frac{de}{dv}=\frac{de}{dt}\frac{dt}{dv}.\end{cases} \tag{8}$$

Now the equation of the areas

$$\frac{dt}{dv}=\frac{r^2}{C} \tag{9}$$

$$=\frac{p^2}{C}(1+e\cos v)^{-2}.$$

Formulas (4), (7) and (9) therefore make it possible to write the values (8) of $da/dv$ and $de/dv$ which, integrated between 0 and $2\pi$ will give the variations of half the major axis and the eccentricity during one revolution.

We may here offer certain hypotheses on medial resistance R. This resistance increases as the velocity; we shall suppose it proportional to a certain power of the velocity V. It varies directly as the distance $r$ from the sun, for the density, and consequently the resistance, of the sun's atmosphere increases inversely as the distance; let us suppose R proportional to a certain power (negative) of $r$. In short let us put

$$R=hV^{\alpha}r^{-\beta}, \tag{10}$$

$h$, $\alpha$ and $\beta$ being positive constants. Since V is proportionate to $\rho$, and $r$ to $1/(1+e\cos v)$, we can write formula (10) as follows:

$$R=k\rho^{\alpha}(1+e\cos v)^{\beta},$$

$k$ being a new positive constant.

In view of these hypotheses on R, the values (8) of $da/dv$ and

$de/dv$, computed by means of the formulas (4), (7) and (9), may be written

$$(11)\qquad \begin{cases} \dfrac{da}{dv} = -a\mathrm{H}(1-e^2)^{-\frac{1}{2}}\rho^{\alpha+1}(1+e\cos v)^{\beta-2}, \\ \dfrac{de}{dv} = -\ \mathrm{H}(1-e^2)^{\frac{1}{2}}\ \ \rho^{\alpha-1}\ (1+e\cos v)^{\beta-2}\ (e+\cos v); \end{cases}$$

where H denotes the positive constant

$$\mathrm{H} = \frac{2p^2k}{na\mathrm{C}};$$

bear in mind that in these values (11)

$$\rho = (1+2e\cos v+e^2)^{\frac{1}{2}}.$$

In order to study the secular variations of $a$ and $e$ we must develop the second members of the values (11) in trigonometric series according to the cosines of the multiples of $v$, and integrate between $v=0$ and $v=2\pi$. By integration all the cosines will be 0; therefore we are interested in the constant terms of these trigonometric developments and especially the sign of these constant terms.

We already know that $da/dv$ is necessarily negative, since $da/dt$ is always negative. Therfore we shall work only with $de/dv$. We must develop in a trigonometric series the expression

$$\rho^{\alpha-1}(1+e\cos v)^{\beta-2}(e+\cos v).$$

Now if we first develop the product of the two first terms we obtain:

$$(12)\qquad \rho^{\alpha-1}(1+e\cos v)^{\beta-2} = \mathrm{A}_0+\mathrm{A}_1\cos v+\mathrm{A}_2\cos 2v+\dots .$$

We observe that $\mathrm{A}_0$ is necessarily positive because it is the mean value of the first member both of whose terms are always positive. Then multiplying the two members of formula (12) by $(e+\cos v)$ we have

$$\rho^{\alpha-1}(1+e\cos v)^{\beta-2}(e+\cos v) = \left(\mathrm{A}_0 e+\frac{\mathrm{A}_1}{2}\right)+\dots ,$$

all the unwritten terms of the second member having their mean value 0.

The second formula (11) therefore gives for the mean value of $de/dv$ during one revolution

$$(13)\qquad \frac{de}{dv} = -\mathrm{H}(1-e^2)^{\frac{1}{2}}\left(\mathrm{A}_0 e+\frac{\mathrm{A}_1}{2}\right).$$

Since the second member of equation (13) is generally negative we conclude from it that the medial resistance has the effect of

diminishing the eccentricity of the orbit. This would be the case particularly whenever $A_1$ is positive. Now according to formula (12) we have

$$A_1=\frac{2}{\pi}\int_0^{\pi}(1+2e\cos v+e^2)^{\frac{\alpha-1}{2}}(1+e\cos v)^{\beta-2}\cos v\,dv.$$

If at the same time

$$\alpha>1, \qquad \beta>2,$$

$A_1$ will be positive, for of two elements of the integral corresponding to the two values $v$ and $\pi-v$ of the variable of integration, one is positive and the other negative, but the positive element possesses a greater absolute value than the negative.

In an analogous way we know that if the two inequalities

$$\alpha>1, \qquad \alpha+2\beta>5,$$

are satisfied, we shall likewise have

$$A_1>0.$$

If we suppose the eccentricity $e$ to be so small that we can disregard its square $e^2$ we shall find more general conditions. The second formula (11) is reduced to

$$\frac{de}{dv}=-H[1+(\alpha-1)e\cos v+(\beta-2)e\cos v](e+\cos v);$$

whence by retaining only the mean value of the second member we derive

$$\frac{de}{dv}=-H\left(e+\frac{\alpha+\beta-3}{2}e\right)$$

$$=-\frac{He}{2}(\alpha+\beta-1).$$

Then in order to diminish the eccentricity it is sufficient that

$$\alpha+\beta>1.$$

In this case even if $\beta=0$ (that is, if the resistance R does not vary with the distance $r$ from the sun) we need only have

$$\alpha>1,$$

that is to say, R increasing more rapidly than the simple power of the velocity. Now we often grant as an approximation that a medial resistance is proportionate to the square of the velocity.

This diminution of the eccentricity because of a medial resistance might have been foreseen in general and without calculation in the following manner. Suppose the resistance is not felt except in the vicinity of the perihelion P (Fig. 1). In that case the planet

undergoes at this point P a sudden diminution of velocity which results in a decrease in the major axis. Since the perihelion remains the same and the aphelion approaches it, it is clear that the eccentricity is lessened. On the other hand, if resistance acts only at the moment of the aphelion, the new orbit would have the same aphelion as the former one, but its perihelion would be nearer that of the sun, and the eccentricity would be increased. In fact the resistance is felt all along the orbit, but two reasons combine to make it felt more strongly at the perihelion: in the first place the velocity is greatest at that point, since the atmosphere which is generally denser nearer the sun offers a greater resistance near the perihelion.

To sum up, the effect of medial resistance on a Keplerian orbit is to diminish both the major axis and the eccentricity.[5] Therefore if we agree with Mr. See that a resisting atmosphere originally extended for vast distances around the sun, we can conceive that a

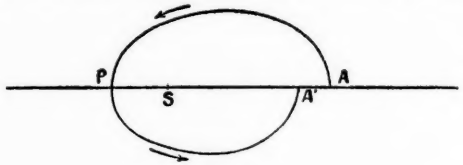


Fig. 1.

body of cosmical origin when passing into the sun's sphere of influence might be able to modify its trajectory. Whether it was parabolic or hyperbolic it now becomes elliptical, because the medial resistance continues to diminish the major axis and the eccentricity of the orbit which approaches the circular form. The resisting atmosphere is gradually absorbed by the sun, and when it finally disappears the smaller body continues to revolve around the sun in its orbit which is almost a circle. Such, according to Mr. See, is the history of all the planets.

Just as the planets have been captured by the sun so also, according to Mr. See, have the satellites been captured by their respective planets.[6]

In order to study this capture we shall take up the comparatively simple case called the restricted problem. The sun S and a planet J

[5] It is easy to recognize that this resistance does not produce any secular effect (at least at the first approximation) on the longitude of the perihelion. To be sure it does not modify the plane of the orbit which retains the same inclination and the same line of nodes with reference to a fixed plane.

[6] *Loc. cit.*, Chap. VIII, pp. 159-182; X, pp 211-236.

(e. g., Jupiter) each revolve around their common center of gravity G in a circular orbit with a constant angular velocity $\omega$ (Fig. 2). It is required to study the motion of a small planet P whose mass is negligible with reference to that of the principal planet J and which consequently will not affect the motion of the latter. We will take as origin the center of gravity G, of the system S – J; as plane of the coordinates $xy$, the plane in which S and J describe their circular orbits; and in this plane rectangular movable axes, the axis of $x$ being the straight line SGJ which connects the sun with Jupiter; the axis of $z$ is the perpendicular to the plane of the orbit at G. The forces acting actually upon the point P $(x, y, z)$ are the attraction of the sun and of Jupiter. These two forces are derived respectively from the two functions of forces[7]

$$U_1=\frac{M}{\rho_1},\ U_2=\frac{M_2}{\rho_2},$$

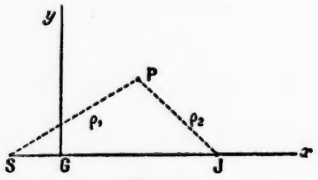


Fig. 2.

$M_1$, $M_2$ being the masses of the sun and Jupiter, $\rho_1$, $\rho_2$ their distances from P. Since the axes are movable we must add to these forces the centrifugal force and the compound centrifugal force. The components of the centrifugal force are

$$\omega^2 x, \qquad \omega^2 y, \qquad 0.$$

The components of the compound centrifugal force are

$$2\omega\frac{dy}{dt}, \qquad -2\omega\frac{dx}{dt}, \qquad 0.$$

Hence the equations of the motion of the planet P with relation to the movable axes are

$$\frac{d^2x}{dt^2}=\frac{dU_1}{dx}+\frac{dU_2}{dx}+\omega^2 x+2\omega\frac{dy}{dt},$$

$$\frac{d^2y}{dt^2}=\frac{dU_1}{dy}+\frac{dU_2}{dy}+\omega^2 y-2\omega\frac{dx}{dt},$$

[7] We assume the mass $m$ of the small planet P to be equal to unity. More exactly, since this mass $m$ is a factor in every case we shall not write it in the formulas.

$$\frac{d^2z}{dt^2}=\frac{dU_1}{dz}+\frac{dU_2}{dz}.$$

If we multiply these three equations

$$dx=\frac{dx}{dt}dt, \qquad dy=\frac{dy}{dt}dt, \qquad dz=\frac{dz}{dt}dt,$$

respectively, and add the results, we obtain a combination immediately integrable which brings us to the following integral

$$\frac{1}{2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right]=\frac{M_1}{\rho_1}+\frac{M_2}{\rho_2}+\frac{\omega^2}{2}(x^2+y^2)-C,$$

known by the name of the integral of Jacobi.

Since the first member of this last equation is positive, the coordinates $x$, $y$, $z$ of the point P will satisfy the inequality

$$\frac{M_1}{\rho_1}+\frac{M_2}{\rho_2}+\frac{\omega^2}{2}(x^2+y^2)-C>0.$$

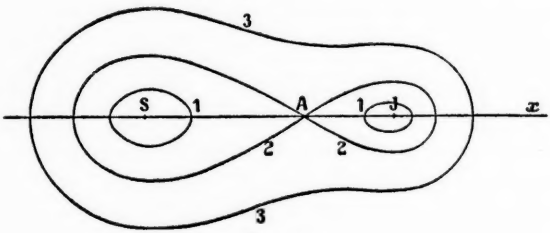


Fig. 3.

Hence the projection $(x, y)$ of the point P on the plane of $xy$ will be within the curve

$$\frac{M_1}{\rho_1}+\frac{M_2}{\rho_2}+\frac{\omega^2}{2}(x^2+y^2)=C;$$

in this equation $\rho_1$ and $\rho_2$ denoting the distances of this projection of the point P from the points S and J. For very great values of the constant C this curve comprises two rings (denoted by 1 on Fig. 3) surrounding the points S and J respectively. As C diminishes, these two rings become dilated and finally unite at a double point A (Curve 2). Then when C is further diminished they finally make only one curve (Curve 3) surrounding at the same time both S and J.[8] Hence when the constant C is not too great the small planet is obliged to remain within Curve 3 but still is free to travel in the proximity either of the sun or of Jupiter. On the contrary

[8] We pay no attention here to certain portions of curves which are very far removed from the origin.

if the constant C is very great the small planet will remain within one of the rings 1; it will be a satellite either of the sun or of Jupiter.

Now the effect of a passive resistance like that of a medium is to increase the constant C of the second member of Jacobi's integral. Hence the curve encircling the small planet constantly contracts. If it was originally Curve 3 at a definite moment it will become Curve 2 with the double point. If at this moment the planet is near the sun it will never return to the proximity of Jupiter; it is captured by the sun. If on the contrary it is in the neighborhood of Jupiter it will never return to that of the sun; it will be captured by Jupiter and from that moment will become one of his satellites.

The theory of Mr. See accounts for the smallness of the eccentricities of the orbits of planets and satellites.[9] But why are the movements of almost all the heavenly bodies in a straight line, and why have their orbits such small mutual inclinations? In the hypothesis of Mr. See these two questions remain without any satisfactory answer. To try to explain the smallness of the inclinations we may suppose that the resisting atmosphere of the sun is of a greatly flattened lenticular form; hence a body whose orbit is greatly inclined to the plane of this disk suffers a resistance much smaller than a body moving in the very plane of the disk. The first body has therefore much less tendency to be captured than the second, and is in the plane of the disk in which the captures of the planets are made.

We may also suppose that the resisting medium itself revolves. It will then tend not to counteract the velocity of the planet revolving within it but to impose upon this planet a certain velocity. Since the resistance is no longer directly opposed to the velocity, the plane of the orbit could vary and tend to diminish its inclination to the equatorial plane of the solar atmosphere.

## FORMATION OF SPIRAL NEBULAS.

In the work previously referred to,[10] Mr. See is concerned with the formation of nebulas, especially with the origin of spiral nebulas.

Let us imagine two masses of cosmical vapor N and N′, almost equal in size and traveling in opposite directions (Fig. 4*a*). As they

[9] The diminution of the eccentricity because of a resisting medium is of first importance not only in the theory of Mr. See; it is taken into consideration also in the theories of Faye and of Du Ligondès.

[10] *Op. cit.*, Chap. XIX.

approach each other their adjacent extremities will be prolonged each in the direction of the other by mutual attraction (Fig. 4*b*) and may even end in uniting to form a single body (Fig. 4*c*) near whose center attraction combined with friction will tend to produce a condensation, a sort of central nucleus. The two masses of vapor N and N′ will turn in the directions of the arrows around this center like two arms of a windmill.

Such, according to Mr. See, would be the origin of the spiral nebulas. The central nucleus would tend to enlarge more and more


Fig. 4.

at the expense of the matter in the two spiral branches N and N′. Hence we see that in the opinion of Mr. See the motion of the matter in the two arms of the spiral nebula contrary to the usual view would be centripetal and not centrifugal. Moreover whether the motion is convergent or divergent the law of areas accounts equally in both cases for the slowness of the arm's revolution around its pivot, that is to say, the spiral form of both arms.

It may happen that the ends of the two masses of vapor N and N′ do not join as they approach each other, but are merely deviated by attraction. Then the phase following phase 2 of Fig. 4. is not

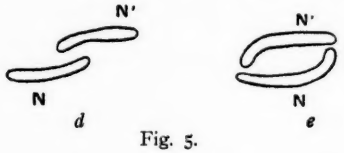


Fig. 5.

phase *c* but phase *d* (Fig. 5) after which it assumes phase *e*. In such a case we have the origin of an annular nebula like that of Lyra. In the two diametrically opposed light portions seen in the ring of Lyra, Mr. See finds an argument for the application of this theory in that adjacent ends of the two masses of vapor N and N′ would not be perfectly united.

Hence Mr. See thinks that an annular nebula is formed by the same mechanical process as spiral nebulas of which it thus proves to be in some sense a particular case. But the annular form is

very rare because the conditions for the formation of a perfect ring are not often realized.

One great objection may be offered to this theory. The two arms of a spiral nebula are usually almost symmetrical. In the ordinary hypothesis in which the movement of the arms is assumed to be divergent this symmetry may be explained by the common origin of the two arms. In the hypothesis of Mr. See there is no way to account for it, for the two masses of cosmical vapor N and N′ which give rise to the nebula and which have met accidentally will not usually be equal. They ought then to give birth to an unsymmetrical nebula.

Mr. See thinks that originally the solar system was a spiral nebula of vast extent. The matter at its center first became agglomerated into particles which with the help of the resistance of the medium were condensed into asteroids, according to the process explained above, and then into planets, which are further increased by bombardment.[11]

Mr. See is led by analogy to believe that the spiral nebulas which are less advanced in their evolution than the solar system are composed of a vast number of very small bodies like the planets or even the moon. If we can not analyze these nebulas it will be because of the extremely small size of their component parts and not because these celestial objects are so excessively remote. Mr. Bohlin has tried to measure the parallax of the nebula of Andromeda (which is a spiral nebula of a continuous spectrum) and he has found it equal to 0″, 17, so that this nebula would be comparatively very near us. But considering how little accuracy the points on the nebulas admit of, can we regard this observation as conclusive and certain?

H. POINCARÉ.

PARIS, FRANCE.

## NOTES ON THE CONSTRUCTION OF MAGIC SQUARES OF ORDERS IN WHICH $n$ IS OF THE FORM $8p+2$.

Referring to the article in the last issue of *The Monist* by Messrs. Andrews and Frierson, under the above heading, it was shown that the minimum series to be used in constructing this class of squares is selected from the series 1, 2, 3,.....$(n+3)^2$, by

[11] Mr. See sees in the lunar craters signs of a bombardment produced at the surface of the moon by the fall of a large number of little satellites. He compares these craters to the marks left by great drops of rain in the mud (*op. cit.*, p. 342, plate XII).

discarding 3 rows and columns from the natural square of the order $n+3$.

It is not necessary, however, to discard the three central rows and columns, as was therein explained, there being numerous variations, the total number of which is always equal to $\left(\frac{n+2}{4}\right)^2$

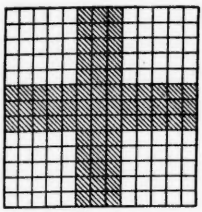

Fig. 1.

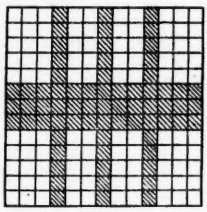

Fig. 2.

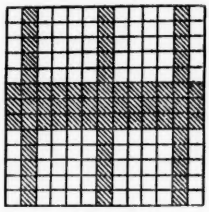

Fig. 3.

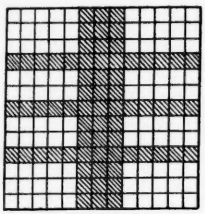

Fig. 4.

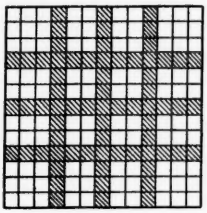

Fig. 5.

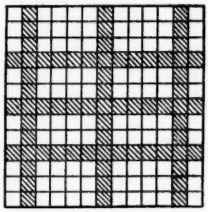

Fig. 6.

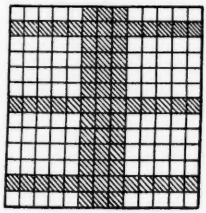

Fig. 7.

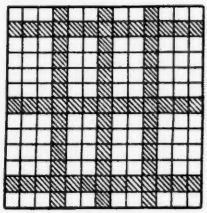

Fig. 8.

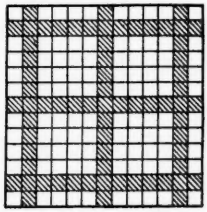

Fig. 9.

therefore the $10^2$ can be constructed with 9 different series, the $18^2$ with 25 different series, the $26^2$ with 49 different series, and so on.

In Figs. 1 to 9 are shown all the possible variations of discarding rows and columns for the $10^2$, Fig. I representing the series explained in the foregoing article.

The central row and column must always be discarded, the remaining two rows and columns can be cast out symmetrically in relation to their parallel central row or column and should be an

odd number of rows or columns from it. In other words, we cast out the central row, then on each side of it we cast out the 1st, 3d,

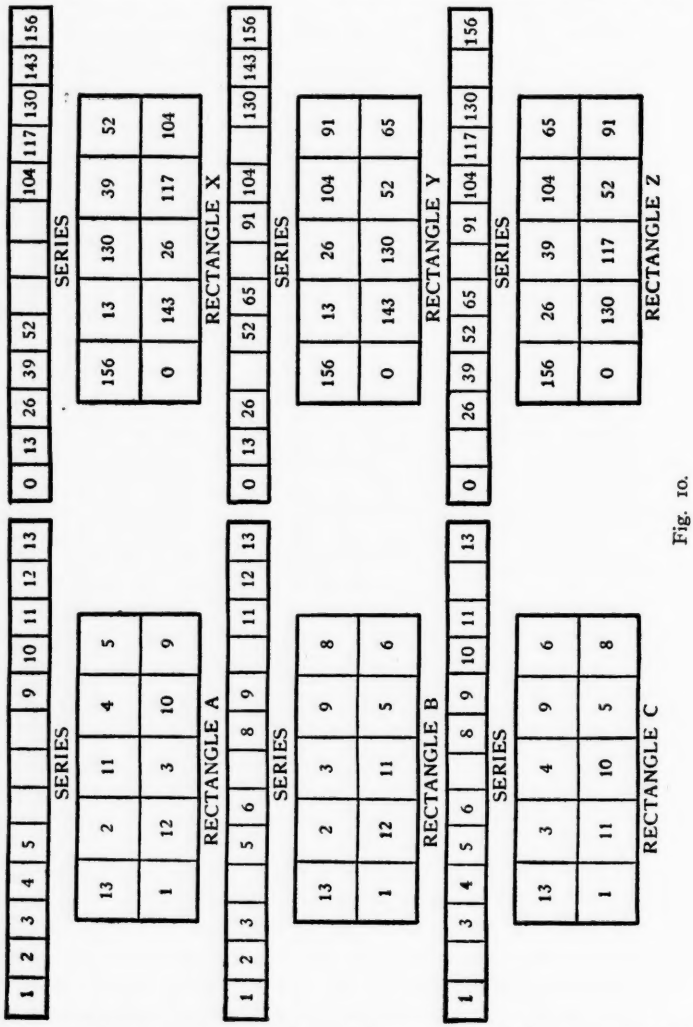


Fig. 10.

5th, or 7th, etc. rows from it, and irrespective of the rows, we do likewise with the columns.

In a manner already explained, numbers are selected according to the series desired and arranged in rectangles with which the magic square is constructed.

A set of rectangles with their respective series is shown in Fig. 10, and the following table will give directions for their use.

| Series | Rectangles (See Fig. 10) |
|---|---|
| Fig. 1 | A and X |
| Fig. 2 | B and X |
| Fig. 3 | C and X |
| Fig. 4 | A and Y |
| Fig. 5 | B and Y |
| Fig. 6 | C and Y |
| Fig. 7 | A and Z |
| Fig. 8 | B and Z |
| Fig. 9 | C and Z |

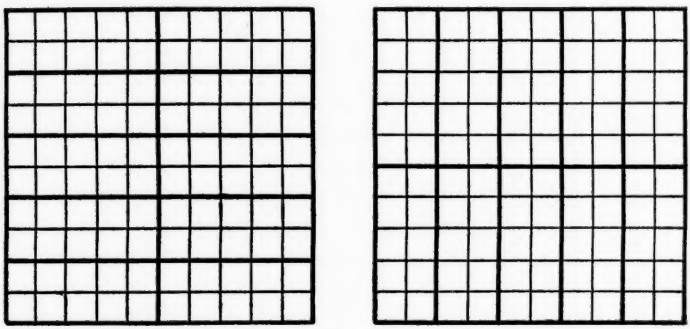

Fig. 11.

For example, suppose we were to construct a square, using the series denoted in Fig. 3. By referring to the table it is seen that we must employ rectangles C and X. By using the La Hireian method these rectangles are placed as shown in Fig. 11, care being taken to arrange them in respect to the final square, whether it is to be associated or non-associated.[1]

A non-associated square resulting from rectangles C and X is shown in Fig. 12. Another example by Mr. Andrews, using the path method is shown in Figs. 13, 14 and 15. Here a series corres-

[1] See preceding article.

ponding to Fig. 8 has been selected and the natural square is shown in Fig. 13, the heavy lines indicating the discarded rows and columns. The rows and columns are re-arranged according to the nu-

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 65 | 107 | 56 | 113 | 58 | 117 | 55 | 108 | 61 | 110 |
| 40 | 128 | 49 | 122 | 47 | 118 | 50 | 127 | 44 | 125 |
| 143 | 29 | 134 | 35 | 136 | 39 | 133 | 30 | 139 | 32 |
| 14 | 154 | 23 | 148 | 21 | 144 | 24 | 153 | 18 | 151 |
| 169 | 3 | 160 | 9 | 162 | 13 | 159 | 4 | 165 | 6 |
| 53 | 115 | 62 | 109 | 60 | 105 | 63 | 114 | 57 | 112 |
| 52 | 120 | 43 | 126 | 45 | 130 | 42 | 121 | 48 | 123 |
| 131 | 37 | 140 | 31 | 138 | 27 | 141 | 36 | 135 | 34 |
| 26 | 146 | 17 | 152 | 19 | 156 | 16 | 147 | 22 | 149 |
| 157 | 11 | 166 | 5 | 164 | 1 | 167 | 10 | 161 | 8 |

Fig. 12.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 13 |
| 27 | 28 | 29 | 31 | 32 | 34 | 35 | 37 | 38 | 39 |
| 40 | 41 | 42 | 44 | 45 | 47 | 48 | 50 | 51 | 52 |
| 53 | 54 | 55 | 57 | 58 | 60 | 61 | 63 | 64 | 65 |
| 66 | 67 | 68 | 70 | 71 | 73 | 74 | 76 | 77 | 78 |
| 92 | 93 | 94 | 96 | 97 | 99 | 100 | 102 | 103 | 104 |
| 105 | 106 | 107 | 109 | 110 | 112 | 113 | 115 | 116 | 117 |
| 118 | 119 | 120 | 122 | 123 | 125 | 126 | 128 | 129 | 130 |
| 131 | 132 | 133 | 135 | 136 | 138 | 139 | 141 | 142 | 143 |
| 157 | 158 | 159 | 161 | 162 | 164 | 165 | 167 | 168 | 169 |

Fig. 13.

merical sequence of the continuous diagonals[1] of rectangles B and Z of Fig. 10, this re-arrangement being shown in Fig. 14.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 11 | 9 | 6 | 13 | 12 | 3 | 5 | 8 |
| 27 | 28 | 37 | 35 | 32 | 39 | 38 | 29 | 31 | 34 |
| 118 | 119 | 128 | 126 | 123 | 130 | 129 | 120 | 122 | 125 |
| 105 | 106 | 115 | 113 | 110 | 117 | 116 | 107 | 109 | 112 |
| 92 | 93 | 102 | 100 | 97 | 104 | 103 | 94 | 96 | 99 |
| 157 | 158 | 167 | 165 | 162 | 169 | 168 | 159 | 161 | 164 |
| 131 | 132 | 141 | 139 | 136 | 143 | 142 | 133 | 135 | 138 |
| 40 | 41 | 50 | 48 | 45 | 52 | 51 | 42 | 44 | 47 |
| 53 | 54 | 63 | 61 | 58 | 65 | 64 | 55 | 57 | 60 |
| 66 | 67 | 76 | 74 | 71 | 78 | 77 | 68 | 70 | 73 |

Fig. 14.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 162 | 1 | 168 | 11 | 161 | 6 | 157 | 12 | 167 |
| 100 | 73 | 104 | 67 | 94 | 74 | 99 | 78 | 93 | 68 |
| 57 | 110 | 53 | 116 | 63 | 109 | 58 | 105 | 64 | 115 |
| 126 | 47 | 130 | 41 | 120 | 48 | 125 | 52 | 119 | 42 |
| 135 | 32 | 131 | 38 | 141 | 31 | 136 | 27 | 142 | 37 |
| 9 | 164 | 13 | 158 | 3 | 165 | 8 | 169 | 2 | 159 |
| 96 | 71 | 92 | 77 | 102 | 70 | 97 | 66 | 103 | 76 |
| 61 | 112 | 65 | 106 | 55 | 113 | 60 | 117 | 54 | 107 |
| 122 | 45 | 118 | 51 | 128 | 44 | 123 | 40 | 129 | 50 |
| 139 | 34 | 143 | 28 | 133 | 35 | 138 | 39 | 132 | 29 |

Fig. 15.

[1] See article in *Monist* of April, 1912.

In constructing the final square, Fig. 15, an advance move – 4, – 5 and a break move 1, 1 was used.

It wiil be unnecessary to show examples of higher orders of these squares, as their methods of construction are only extensions of what has been already described. It may be mentioned that these squares when non-associated can be transformed into associated squares by the method given in Messrs. Andrews and Frierson's article.

HARRY A. SAYLES.

SCHENECTADY, N. Y.

## POSTSCRIPT ON BUDDHISM AND CHRISTIANITY.

My article on the "Contributions of Buddhism to Christianity," which appeared in *The Monist* of October, 1911, called forth two criticisms in the following number (January 1912). One was by Albert J. Edmunds, "Buddhist Loans to Christianity," pp. 129 ff., and the other by Wilfred H. Schoff, "First Century Intercourse Between India and Rome," pp. 138 ff.

Even before these criticisms reached me, I began to doubt whether my standpoint that Buddhist influences were "not yet to be found in the canonical Gospels, but first in the Apocryphal Gospels," could be maintained in this categorical form.[1] The historical *possibility* for the infiltration of Buddhist material into the canonical Gospels I have never denied, but only its *probability*. I take pleasure in using this opportunity to grant that by the lucid critique of Edmunds the probability of the hypothesis of Buddhist loans in the New Testament has increased in my opinion.

The connection of the Asita-Simeon parallel with the praise of the heavenly hosts in both the Suttanipāta and in the Gospel of Luke has strongly impressed me even though I can not concede to Edmunds that this connection is an "organic" one on both sides. The connection is organic only in the Pali source and not in Luke, where in the second chapter the Simeon story does not stand in an intrinsic connection with the angelic hymn but only *near* it. But even this correspondence is certainly remarkable enough.

The exposition which Edmunds has given of the temptation parallels (Samyuttanikāya and Luke iv. 1-2) also decidedly increases the probability of the loan hypothesis. Because of this the Buddhist origin of some other New Testament stories, to which I have heretofore only with hesitancy granted a remote possibility that they

[1] See my article, "Buddhistisches im Neuen Testament," in *Das Freie Wort*, Frankfort, December 1911, pp. 674 ff.

might have been borrowed from India, also becomes of course more probable.

Edmunds is entirely mistaken in his explanation of the Wandering Jew (pp. 137-138).[2] Mark ix. 1: "Verily I say unto you, There be some here of them that stand by, who shall in no wise taste of death, till they see the kingdom of God come with power," does not in the least contain the germ of this legend but simply expresses like the other passages on the Parousia (Matt. x. 23; xvi. 28; Luke ix. 27) the conviction of Jesus that the end of the world was at hand.

The article of W. H. Schoff elucidates in a clear exposition well-known facts about the commercial intercourse between India and the Occident in the first century after Christ, but he brings no positive proof that an exchange of ideas necessarily went hand in hand with the extensive commercial intercourse. Especially, he, as the translator of the Periplus, ought to have inferred from this text that the mariners and traders of those days had but little thought for anything but their merchandise. The author of the Periplus, who describes his journey to India between 70 and 75 A. D., treated only of what would be interesting to the merchant and mariner, but otherwise shows that he was uninformed about the most commonplace things and says not one word about religion. Likewise the Indian merchants who had settled in Alexandria were according to the testimony of Dio Chrysostom (*Orat.* 35) ignorant people and probably of Dravidian race. They would have taken no more interest in religious questions than the Greek or Roman merchants of their time.

When Schoff (page 141) describes the merchants as "bearing ideas no less than goods," this is simply begging the question.

More important for our purpose than all reports of ancient commercial relations seems to me the observation of Max Müller expressed in the following words:[3] "Though we have no tangible evidence of anything like translations, whether Oriental or Occidental, at that time, we seem perfectly within our right when we look upon the numerous coincidences between the fables of Æsop and the fables occurring in Sanskrit and Pāli literature as proving the fact that there was a real literary exchange between India, Persia, Asia Minor and Greece beginning with the 6th century B. C."

TÜBINGEN, GERMANY. R. GARBE.

[2] Compare also *Buddhist and Christian Gospels*, 4th ed., II, pp. 264 ff.

[3] In the article "Coincidences" in *Last Essays*, I, 269-270.

## POINCARE'S COSMOGONIC HYPOTHESES.

Prof. H. Poincaré has just published an important book[1] which treats the interesting problem of the origin of the world according to the scientific views of modern philosophers and naturalists. Professor Poincaré in the first chapter discusses Kant's hypothesis and subjects it to a critical analysis. The second chapter is devoted to La Place; the third analyzes La Place's hypothesis and discusses the work of La Roche, especially the theory of the stability of rings and the formation of satellites. Subdivisions of this third chapter treat the hypothesis of a uniform notation, the rings of Saturn, the rupture of rings according to La Place and the formation of planets and satellites, and the author sums up the objections to the theory of La Place.

The fourth chapter is devoted to the hypothesis of H. Faye, according to which the earth is much older than the sun. Chapter five discusses the hypothesis of du Ligondès who claims that Kant's hypothesis stand in contradiction to the principle of the gases. The sixth chapter treats the hypothesis of Prof. T. J. J. See, which will be of special interest to American readers because he is a native American and is the astronomer of the Naval Observatory, Mare Island, California. This chapter together with the thirteenth is reproduced in an English translation on another page of this issue. The seventh chapter discusses the theory of Sir George Howard Darwin, his theory of tides, especially the internal tides of the earth, the accelerative influence of cooling down, and his hypothesis of the formation of the moon. The eighth chapter treats the theory of solar and terrestrial heat, as well as the adiabatic equilibrium of a perfect gas.

Chapter nine treats of the theory of Sir Norman Lockyer, Chapter ten of Schuster and Chapter eleven of Arrhenius's theories: Chapter twelve compares the mass of the Milky Way with a gaseous mass. Its substance is comparable to the radiant matter of Krookes, rather than to a true gas. He then treats possible causes of the flattening of the Milky Way and concludes with a consideration of the star clusters of Kapteyn and Schiaparelli.

In the thirteenth chapter our author returns to Professor See and discusses his view of the formation of the nebular spirals; and the last chapter is devoted to the hypothesis of Emile Belot. P. C.

[1] *Leçons sur les hypothèses cosmogoniques.* Paris: Hermann, 1911. Price 12 francs.